Review on functions II

Name: ______________

Math 1110 - Instructor: Itamar Oliveira

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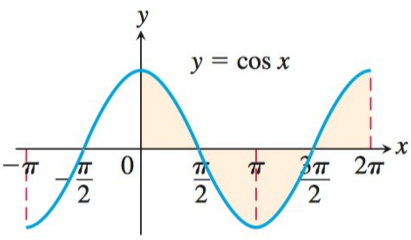
1 More on trigonometric functions

(1) Write down the definition of periodic function. Complete the blanks in the picture below with the domain, range and period of each function.

(2) If $x \in \left[\pi, \frac{3\pi}{2}\right]$ and $\sin x = -\frac{1}{2}$, compute $\cos x$.

(3) Is $f(x) = \sin\left(10e^{\sqrt{\pi}}x\right)$ periodic?



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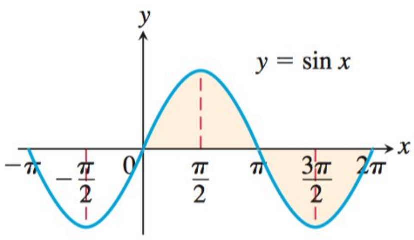

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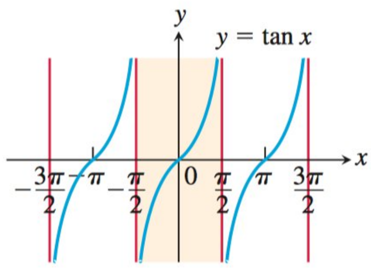

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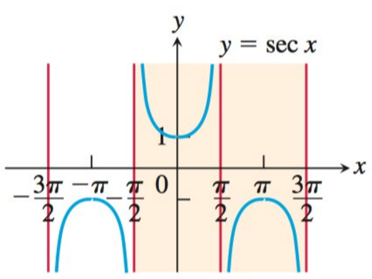

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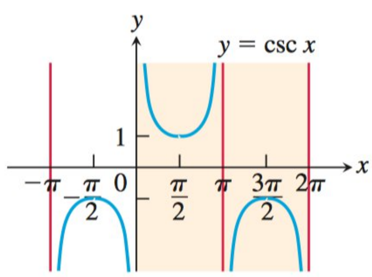

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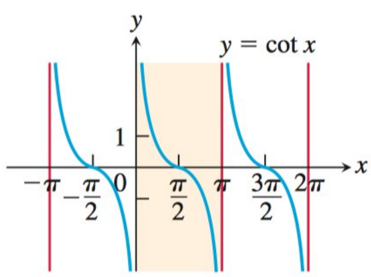

Domain:
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2 Inverse function

Definition 1. A function $f(x)$ is **one-to-one** (or **injective**) on a domain D if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ in D.

(1) Is $f(x) = \sin x$ one-to-one? (1)

(2) Which functions from your pre-class activity are one-to-one?

(3) Horizontal line test for one-to-one functions:

Definition 2. Let f be a one-to-one function on a domain D with range R. The **inverse function** f^{-1} is defined by

$$f^{-1}(b) = a \quad \text{if} \quad f(a) = b.$$

The domain of f^{-1} is R and the range of f^{-1} is D.

Be careful! The "-1" in f^{-1} is *not* an exponent: $f^{-1}(x)$ **does not mean** $1/f(x)$.

(1) Identify the domain and find the inverse of the functions below:

(a) $f(x) = \dfrac{x+3}{x-2}$.

(b) $f(x) = x^2 - 2x$, $x \leq 1$.

3 Logarithms

Definition 3. The **logarithm function with base** a, written $y = \log_a x$, is the inverse of the base a exponential function $y = a^x$ ($a > 0$, $a \neq 1$).

Special case: if the base is e, $y = \log_e x$ is written as $y = \ln x$ and is called the *natural logarithm function.*

1. Simplify the quantities $2 \ln \sqrt{e}$ and $\ln(\ln e^e)$ using the properties that you wrote.

2. Find two different expressions of $x = \ldots$ using the natural logarithm and the exponential function. When are these expressions true (meaning, for which x)?

3. Here we have a question involving many things that we covered in class. Suppose $f(x)$ has inverse

$$f^{-1}(y) = \ln\left(\frac{y^2}{4}\right)$$

and the domain of f^{-1} is $(-\infty, 0)$. Find the function $f(x)$ and state its domain and range.