Rates of change and limits (part I) Name: ____________

Math 1110 - Instructor: Itamar Oliveira January 29, 2020

1 Rates of change and tangent to curves

A big part of Calculus is devoted to study how functions change over time. A more precise way of quantifying that will be presented soon, but today we take the first step in that direction.

Definition 1. The average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}, \quad h \neq 0.$$

(1) Consider the function $f : [0, 1] \to \mathbb{R}$ given by $f(x) = 1$ for all $x \in [0, 1]$. What does this definition say about f? (1)

(2) Explain how you get the *tangent* line to a curve at a point by using secants.

(3) Example to work in class: find the slope of $y = x^3$ at the point $P = (2, 8)$.

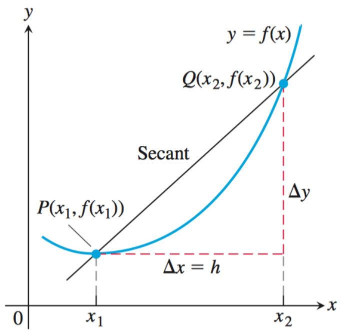

Figure 1: Rate of change.

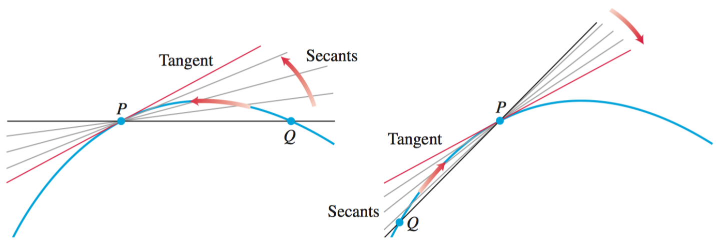

Figure 2: Tangent.

2 Limits (Part I)

(1) How does the function

$$f(x) = \frac{x^2 - 4x + 4}{x - 2}$$

behave near 2?

(2) Motivated by the previous example, write down what the following means:

$$\lim_{x \to a} f(x) = L.$$

Also determine L in the case $a = 2$.

Remark 2. A formal definition of limit will be presented later, but we will try to understand it through examples first.

1. Examples of functions for which "limits always exist" (and their graphs):

2. Now we examine some examples of what could go wrong for a limit not to exist. Three main things can prevent a function from having a limit at a point. Use the space below to explain why each function in the picture does not have a limit as x approaches 0.

   (a) The function *jumps.*

   (b) The function *grows too large to have a limit.*

   (c) The function *oscillates too much to have a limit.*

Remark 3. A combination of the phenomena above also prevents a function from having a limit: for example, it could be the case that a function grows too large **and** oscillates a lot close to some point.