LIMITS (PART III) NAME: ____________

Math 1110 - Instructor: Itamar Oliveira February 3, 2020

# 1 The sandwich (squeeze) theorem

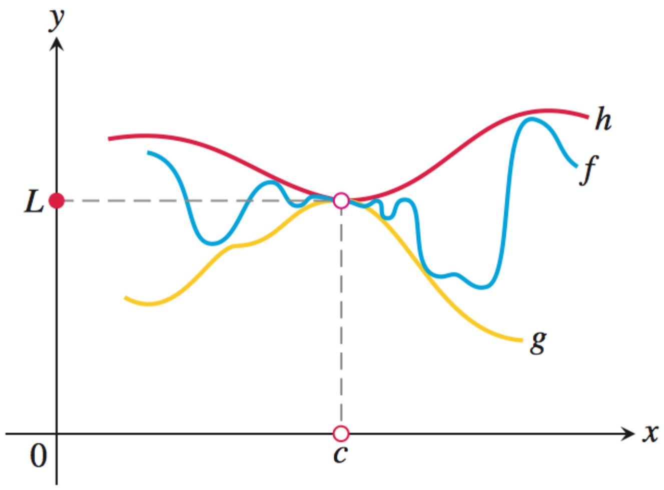
**Theorem 1.** *Suppose that $g(x) \le f(x) \le h(x)$ for all $x$ in some open interval containing $c$, except possibly at $x = c$ itself. Suppose also that*

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L.$$

*Then* $\lim_{x\to c} f(x) = L$.

(1) If $2 - x^2 \le g(x) \le 2\cos x$ for all $x$, find $\lim_{x\to 0} g(x)$.

(2) Compute $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right)$.



## 2 One-sided limits

To have a limit $L$ as $x$ approaches $c$, a function $f$ must be defined on both sides of $c$ and its values $f(x)$ must approach $L$ as $x$ approaches $c$ from either side.

Notation for the right-hand limit: $\boxed{\phantom{xxxxxxxx}}^{(1)}$.

Notation for the left-hand limit: $\boxed{\phantom{xxxxxxxx}}^{(2)}$.

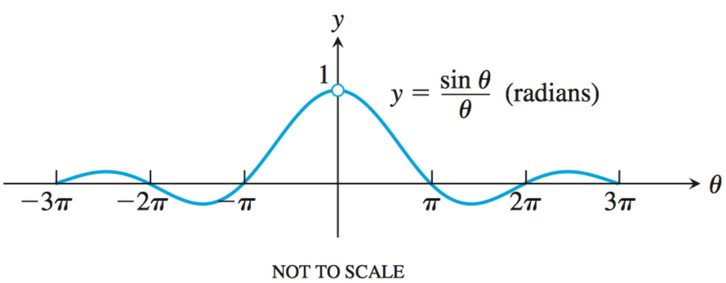
(1) Draw the graph of

$$f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1. \end{cases}$$

Find $\lim_{x\to 1^+} f(x)$ and $\lim_{x\to 1^-} f(x)$. Does $\lim_{x\to 1} f(x)$ exist? If so, why? It not, why not?

(2) Now we compute one of the most famous limits out there (known as the *fundamental limit*):

$$\lim_{x\to 0} \frac{\sin x}{x}.$$



(3) Compute the following limits:

(a) $\lim_{y \to 0} \frac{\sin 3y}{4y}$.

(b) $\lim_{h \to 0} \frac{\cos h - 1}{h}$.

(4) Let

$$f(x) = \begin{cases} x^2 \sin(1/x), & x < 0 \\ \sqrt{x}, & x > 0. \end{cases}$$

Does $\lim_{x \to 0} f(x)$ exist? Justify.