Continuity **Name:** ____________

Math 1110 - Instructor: Itamar Oliveira February 5, 2020

1 Definition and examples

Definition 1. Let $c \in \mathbb{R}$. A function f is **continuous at** c if ____________ (1).

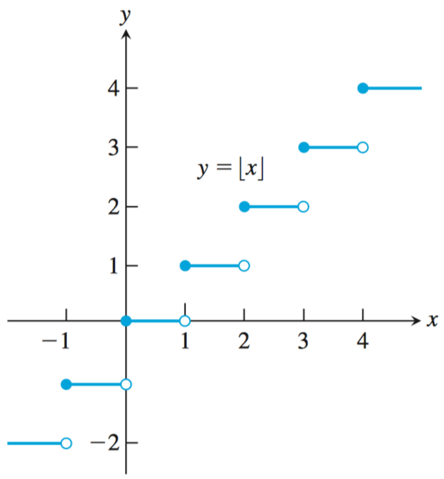
It is **right-continuous at** c (or continuous from the right) if ____________ (2).

It is **left-continuous at** c (or continuous from the left) if ____________ (3).

We say that f is **continuous** if it is continuous at every point of its domain.

1. Look at the graph below and discuss the continuity of the function at integers.

2. Which functions from your pre-class activity are continuous?



3. The picture below represents the graph of function $q(x)$. For parts (a) – (c), circle all listed values satisfying the given statement. If there are no such values, circle NONE.

(a) For which of the following values of a does $\lim_{x\to a} q(x)$ exist?

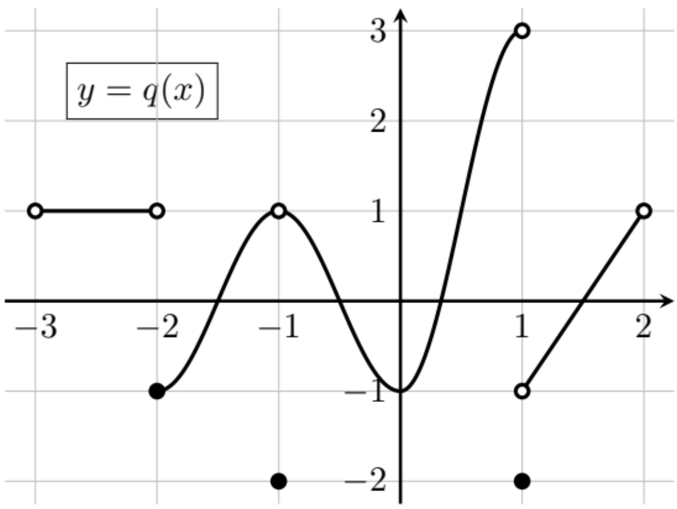
$a = -2$ $a = -1$ $a = 0$ $a = 1$ NONE

(b) For which of the following values of b is $q(x)$ continuous at $x = b$?

$b = -2$ $b = -1$ $b = 0$ $b = 1$ NONE

(c) For which of the following values of c does $\lim_{x\to c^+} q(x) = q(c)$?

$c = -2$ $c = -1$ $c = 0$ $c = 1$ NONE



2 Properties of continuous functions

When determining if some function is continuous or not, there are many tools at our disposal. What follows is a consequence of the properties of limits that we saw:

Theorem 2. *If the functions f and g are continuous at $x = c$, then the following combinations are also continuous at $x = c$:*

(a) $f + g$ *and* $f - g$.

(b) $k \cdot f$, *for any number* k.

(c) $f \cdot g$.

(d) f/g, *provided* $g(c) \neq 0$.

(e) f^n, n *a positive integer.*

(f) $\sqrt[n]{f}$, *provided it is defined on an open interval containing* c, *where* n *is a positive integer.*

Theorem 3. *If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.*

As a consequence of the last theorem, write down how we can "pass the limit inside" when dealing with compositions of continuous functions:

1. Use continuity to compute the following limits:

   (a) $\lim_{x \to \pi} \sin(x - \sin x)$.

   (b) $\lim_{t \to 0} \sin\left(\frac{\pi}{2} \cos(\tan t)\right)$.

3 Another example of what you can be asked to do

A very common problem involving continuity is the one about finding the right parameters that will make a function continuous. Here's an example:

1. For what values of a and b is

$$f(x) = \begin{cases} -2 & \text{if } x \leq -1 \\ ax - b & \text{if } -1 < x < 1 \\ 3 & \text{if } x \geq 1 \end{cases}$$

   continuous at every x?