Asymptotes and tangent lines

Name: _____

Math 1110 - Instructor: Itamar Oliveira

February 12, 2020

1 More on horizontal asymptotes

The Sandwich Theorem also holds for limits as $x \to \pm\infty$! For example, use it to find the horizontal asymptote of the function

$$f(x) = 2 + \frac{\sin x}{x}.$$

Some weird things can happen when dealing with infinite limits. For example, try to compute

$$\lim_{x\to\infty} \left(x - \sqrt{x^2+16}\right).$$

Now compute

$$\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right).$$

2 Infinite limits and vertical asymptotes

It can happen that a function "blows up" when x approaches some real number a. For example, look at the graph of $f(x) = 1/x$ in the picture below. For this function, we write

$$\lim_{x\to 0^+} f(x) = \infty \qquad \text{and} \qquad \lim_{x\to 0^-} f(x) = -\infty.$$

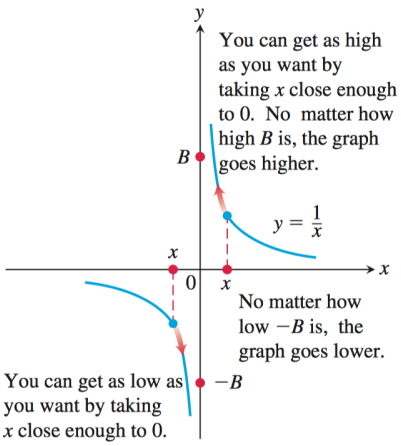
The y-axis (also known as the line $x = 0$) is a *vertical asymptote* of the graph of $f(x) = \frac{1}{x}$. In general,

Definition 1. A line $x = a$ is a **vertical asymptote** of the graph of the function $y = f(x)$ if either

$$\lim_{x\to a^+} f(x) = \pm\infty \qquad \text{or} \qquad \lim_{x\to a^-} f(x) = \pm\infty.$$

1. Find the horizontal and vertical asymptotes of the graph of

$$f(x) = -\frac{8}{x^2 - 4}.$$



3 Some questions

1. Suppose that $f(x)$ and $g(x)$ are polynomials in x and that

$$\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right) = 2.$$

Can you conclude anything about $\lim_{x\to-\infty}\left(\frac{f(x)}{g(x)}\right)$? Justify.

2. Compute

$$\lim_{x\to\infty}\frac{x+\sin x+2\sqrt{x}}{x+\sin x}.$$

4 Finding the tangent to the graph at a point

Definition 2. The **slope of the curve** $y = f(x)$ at the point $P(x_0, f(x_0))$ is the number

$$m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \qquad \textit{(provided the limit exists).}$$

The **tangent line** to the curve at P is the line through P with this slope.

1. Find an equation for the tangent to $y = (x - 1)^2 + 1$ at the point $(1, 1)$ and sketch it.