Differentiation rules Name: ____________

Math 1110 - Instructor: Itamar Oliveira February 17, 2020

1 Rules that simplify many computations

Write down the proof the statements inside each of the following boxes, as well as the examples given in class.

If f has a constant value $f(x) = c$, then

$$\frac{df}{dx}(x) = 0.$$

If n is a positive integer and $f(x) = x^n$, then

$$\frac{df}{dx}(x) = \frac{d}{dx}x^n = nx^{n-1}.$$

Remark 1. This is actually true for any real number n.

If u is a differentiable function of x, and c is a constant, then

$$\frac{du}{dx}(x) = \frac{d}{dx}(cu) = c\frac{du}{dx}.$$

Derivative sum rule: If u and v are differentiable functions of x, then their sum $u + v$ is differentiable at every point where u and v are both differentiable. At such points,

$$\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}.$$

1. Find the derivative of the polynomial $y = x^3 + \frac{4}{3}x^2 - 5x + 1$.

2. Does the graph of $f(x) = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where?

Derivative product rule: If u and v are differentiable functions at x, then so is their product $u \cdot v$, and

$$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}.$$

Remark 2. In *prime notation*, $(uv)' = uv' + vu'$. In *function notation*,

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x).$$

1. Find the derivative of $y = (x^2 + 1)(x^3 + 3)$.

Derivative quotient rule: If u and v are differentiable at x and if $v(x) \neq 0$, then the quotient u/v is differentiable at x, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}.$$

Remark 3. In function notation,

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}.$$

1. Find the derivative of

$$y = \frac{t^2 - 1}{t^3 + 1}.$$