More on derivatives — Name: ____________

Math 1110 - Instructor: Itamar Oliveira — February 19, 2020

1 The derivative as rate of change

Definition 1. The **instantaneous rate of change** of a function f with respect to x at x_0 is the derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \tag{1}$$

provided the limit exists.

When the function $f(t)$ represents the position of an object as a function of time on a coordinate axis, we will use a particular vocabulary borrowed from Physics:

1. The **displacement** of the object over the time interval $[t, t + \Delta t]$ is

$$\Delta s = f(t + \Delta t) - f(t).$$

2. The **average velocity** of the object over $[t, t + \Delta t]$ is

$$v_{av} = \frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

3. The **velocity (instantaneous velocity)** is the derivative of position with respect to time:

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

   Remark: besides telling how fast an object is moving, the velocity tells the direction of motion. If it is *positive*, the object is moving forward. If it is *negative*, it is moving backwards.

4. Speedometers don't care about direction, they just tell you how fast you are going. Associated to this we have the concept of **speed**: it is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left| \frac{ds}{dt} \right|.$$

   Don't forget:

   Velocity is not the same as Speed

5. **Acceleration** is the derivative of velocity with respect to time.

$$a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt^2}.$$

6. **Jerk** is the derivative of acceleration with respect to time:

$$j(t) = \frac{da}{dt} = \frac{d^3 s}{dt^3}.$$

1. A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ ft after t sec.

   (a) How high does the rock go?

   (b) What are the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?

   (c) What is the acceleration of the rock at any time t during its flight (after the blast)?

   (d) When does the rock hit the ground again?

2. The graph below shows the velocity $v = ds/dt = f(t)$ (m/sec) of a body moving along a coordinate line.

   (a) When does the body reverse direction?

   (b) When (approximately) is the body moving at a constant speed?

   (c) Graph the body's speed for $0 \leq t \leq 10$.

   (d) Graph the acceleration, where defined.