Linearization and differentials

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1 Introduction and examples

Definition 1. If f is differentiable at $x = a$, then the approximating function

$$L(x) = f(a) + f'(a)(x - a)$$

is the **linearization** of f at a. The approximation

$$f(x) \approx L(x)$$

of f by L is the **standard linear approximation** of f at a. The point $x = a$ is the **center** of the approximation.

1. Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 3$.

Definition 2. Let $y = f(x)$ be a differentiable function. The **differential** dx is an independent variable. The **differential** dy is

$$dy = f'(x)dx.$$

Unlike the independent variable dx, the variable dy is always a dependent variable. It depends on both x and dx. If dx is given a specific value and x is a particular number in the domain of the function f, then these values determine the numerical value of dy. Often the variable dx is chosen to be Δx, the change in x.

2. Find dy if $y = x^5 + 37x$.

3. Look at the picture below and explain what the differential dy means geometrically.

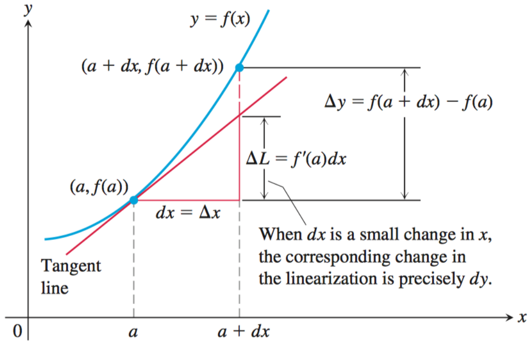
Remark 3. We sometimes write

$$df = f'(x)dx$$

in place of $dy = f'(x)dx$, calling df the differential of f.

Remark 4. Differentiation formulas also hold for df! Compute $d(\tan 2x)$ and $d(x/(x+1))$.

4. Use differentials to estimate $(7.97)^{1/3}$ and $\sin\left(\frac{\pi}{6} + 0.01\right)$.



Remark 5. If $y = f(x)$ is differentiable at $x = a$ and x changes from a to $a + \Delta x$, the change Δy in f is given by

$$\Delta y = f'(a)\Delta x + \varepsilon \Delta x,$$

in which $\varepsilon \to 0$ as $\Delta x \to 0$.

5. The radius r of a circle increases from $a = 10m$ to $10.1m$. Use dA to estimate the increase in the circle's area A. Estimate the area of the enlarged circle and compare your estimate to the true area found by direct calculation. What is the error?

6. You want to calculate the depth of a well from the equation $s = 16t^2$ by timing how long it takes a heavy stone you drop to splash into the water below. How sensitive will your calculations be to a 0.1-sec error in measuring the time?