Textbook Exercises:

6.5 (a) Denote the probabilities of winning, drawing and losing by $p_1$, $p_2$ and $p_3$ respectively. Then under neutral risk, we will have

$$
\begin{align*}
(120 - 100)p_1 + (110 - 100)p_2 + (84 - 100)p_3 &= 0 \\
(30 - 50)p_1 + (55 - 50)p_2 + (60 - 50)p_3 &= 0 \\
p_1 + p_2 + p_3 &= 1
\end{align*}
$$

$$
\Rightarrow \begin{cases} 
    p_1^* = 0.3 \\
    p_2^* = 0.2 \\
    p_3^* = 0.5 
\end{cases}
$$

So the option price to buy Netscape for $50 is

$$
c = p_1^* \cdot 0 + p_2^* \cdot 5 + p_3^* \cdot 10 = 56
$$

(b) Denote the amount of cash, the shares of Microsoft stock to buy/sell, and the shares of Netscape stock to buy/sell by $x$, $y$, $z$ respectively. Then

<table>
<thead>
<tr>
<th>Cash</th>
<th>MS</th>
<th>NS</th>
<th>Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>win</td>
<td>1</td>
<td>20</td>
<td>-20</td>
</tr>
<tr>
<td>draw</td>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>lose</td>
<td>1</td>
<td>-16</td>
<td>10</td>
</tr>
</tbody>
</table>

If we can replicate the option

$$
\begin{align*}
x + 20y - 20z &= 0 \\
x + 10y + 5z &= 5 \\
x - 16y + 10z &= 10
\end{align*}
$$

$$
\Rightarrow \begin{cases} 
    x = 6 \\
    y = -\frac{1}{6} \\
    z = \frac{2}{15}
\end{cases}
$$
Therefore, it's verified that the option price should be 6.

Notice that $V_3 = (A_3 - 4)$. 

$V_2(HH) = 0.4 \left( V_3(HHH) + V_3(HHT) \right) = 0.4 \left( 11 + 5 \right) = 6.4$

$V_2(HT) = 0.4 \left( V_3(HTH) + V_3(HTT) \right) = 0.4 \left( 2 + 0.5 \right) = 1$

Likewise, we can fill in $V_2(TH) = 0.2$, $V_2(TT) = 0$

$V_1(H) = 0.4 \left( V_2(HH) + V_2(HT) \right) = 2.96$

$V_1(T) = 0.4 \left( 0.2 + 0 \right) = 0.08$

$V_0 = 0.4 \left( 2.96 + 0.08 \right) = 1.216$

$\Delta_2(HH) = \frac{11 - 5}{32 - 8} = \frac{1}{4}$

$\Delta_2(HT) = \frac{2 - 0.5}{8 - 2} = \frac{1}{4}$

$\Delta_2(TH) = \frac{0.5 - 0}{8 - 2} = \frac{1}{12}$

$\Delta_2(TT) = 0$
\[
\Delta_1 (H) = \frac{6.4 - 1}{10 - 4} = \frac{9}{20}
\]
\[
\Delta_1 (T) = \frac{0.2 - 0}{4 - 1} = \frac{1}{15}
\]
\[
\Delta_0 = \frac{2.96 - 0.08}{8 - 2} = \frac{12}{25}
\]

(b) \[V_0 = \mathbb{E}^* \left[ \frac{V_3}{(1+r)^3} \right] = \left( \frac{4}{5} \right)^3 \cdot \frac{1}{8} (11 + 5 + 2 + 0.5 + 0.5 + 0 + 0 + 0) = 1.216\]

Additional Problem:

(a) \[V_p - V_c = \frac{K}{1+r} - S_0\]

\[\Rightarrow V_p = \frac{K}{1+r} - S_0 + V_c \geq 0\]

\[\Rightarrow V_c \geq S_0 - \frac{K}{1+r}\]

\[> S_0 - K \text{ since } \frac{1}{1+r} < 1\]

(b) If we exercise at time 0, we get \[\max \{ k - S_0, 0 \} \]

If we exercise at time 1, we get \(V_p\).

- Suppose \( r = 0 \):

  when \( K \leq S_0 \), \( \max \{ k - S_0, 0 \} = 0 \)

  \[V_p = V_c + k - S_0 \geq 0\] \( \text{ Optimal at time 1.}\)

  when \( K > S_0 \), \( V_p = V_c + k - S_0 \geq k - S_0 = \max \{ k - S_0, 0 \} \)

  So it's optimal to hold until time 1 as well.
Now consider $r > 0$. Then it's not guaranteed that $V_p > K - S_0$ (we only have $V_p \geq \frac{K}{1+r} - S_0$). Thus the argument breaks down. However, in part (a) $\frac{1}{1+r} \leq 1$ is always true so the result in part (a) still holds regardless of the value of $r$. 