Textbook exercises (from section 6.8): 6.5, 6.8. In exercise 6.8, to compute the replicating portfolio, use formula (6.13) from section 6.2. If you are not sure how to solve this problem, look at examples 6.2 and 6.3 from section 6.3.

Additional problem:

1. Recall the put-call parity formula (Theorem 6.6): The values $V_P, V_C$ of European put and call options with the same strike price $K$ and expiration time $N$ are related by

$$V_P - V_C = \frac{K}{(1+r)^N} - S_0,$$

where $r$ is the interest rate and $S_0$ is the stock price at time 0. In this problem we will assume that $N = 1$.

(a) Suppose you hold an American call option with strike $K$. Its value is max{$S_0 - K, 0$} if you exercise now or $V_C$ if you wait until time 1, so the overall value is whichever of those two payoffs is greater. Clearly if $S_0 < K$ then it is optimal to hold the option until time 1, since $V_C \geq 0$. If $S_0 \geq K$, use the put-call parity formula along with the fact that $V_P \geq 0$ to demonstrate that $V_C \geq S_0 - K$, which implies that it is still optimal to hold the option until time 1.

(b) When the interest rate is $r = 0$, use a similar argument as in part (a) to prove that it is also optimal to hold an American put option with strike $K$ until time 1. When $r > 0$, explain why this argument breaks down while the reasoning you used in (a) is still valid.