
Additional problems:

1. Here is the transition matrix for a Markov chain on the state space \( \{1, 2, 3, 4, 5\} \):

\[
P = \begin{bmatrix}
0.2 & 0.8 & 0 & 0 & 0 \\
0.8 & 0.2 & 0 & 0 & 0 \\
0.3 & 0.1 & 0 & 0 & 0.6 \\
0 & 0 & 0 & 0.4 & 0.6 \\
0 & 0 & 0 & 0.3 & 0.7
\end{bmatrix}
\]

(a) List the communicating classes of recurrent states. For each communicating class \( C_i \), let \( \pi_i \) be the stationary distribution for \( P \) that is supported on \( C_i \) (meaning that \( \pi_i(x) = 0 \) for all \( x \not\in C_i \)). Use formula (1.8) from section 1.4 of the textbook to determine each \( \pi_i \).

(b) Characterize all the stationary distributions of the Markov chain.

(c) Let \( \mu = [a \ b \ c \ d \ e] \) be a probability distribution on the state space. Provide a formula for \( \lim_{n \to \infty} \mu P^n \) in terms of \( a, b, c, d, e \). Use the behavior of the Markov chain to explain why this formula is correct.

(d) Suppose the third row of \( P \) is replaced with \([0.3 \ 0 \ 0.1 \ 0 \ 0.6]\). Repeat part (c), again explaining the formula using your understanding of the Markov chain.

2. Let \( P \) be the transition matrix for a Markov chain. Suppose \( \lim_{n \to \infty} P^n \) exists, and denote this limiting matrix by \( P^\infty \). Prove that every row of \( P^\infty \) is a stationary distribution for the Markov chain. \textit{Hint:} What is \( P^\infty P \)?