Math 6710, Fall 2016
Homework 11

1. Durrett Exercise 3.3.2, part (i) only.

2. Durrett Exercise 3.3.10.

3. Durrett Exercise 3.3.14.

4. Durrett Exercise 3.4.3.

5. Let $X_1, X_2, \ldots$ be independent random variables such that each X_n is uniformly distributed on $[-n, n]$. Let $S_n = X_1 + \cdots + X_n$. Find α, μ, and σ^2 such that $n^{-\alpha} S_n$ converges weakly to a normal random variable with mean μ and variance σ^2.

Hint: $1 + s \approx e^s$ if s is close enough to zero. Similarly,

$$\prod_{k=1}^{n} (1 + s_k) \approx \prod_{k=1}^{n} e^{s_k} = \exp\left(\sum_{k=1}^{n} s_k\right)$$

if the s_k are close enough to zero. Suppose that for each n we choose a different list $s_{n,1}, \ldots, s_{n,n}$, so that

$$\prod_{k=1}^{n} (1 + s_{n,k}) \approx \prod_{k=1}^{n} e^{s_{n,k}} = \exp\left(\sum_{k=1}^{n} s_{n,k}\right).$$

If we have

$$\lim_{n\to\infty} \sum_{k=1}^{n} s_{n,k} = \lambda,$$

then Fact 11.2 in the notes provides sufficient conditions to conclude that

$$\lim_{n\to\infty} \prod_{k=1}^{n} (1 + s_{n,k}) = e^{\lambda}$$

as one might expect.