Math 6710, Fall 2016
Homework 5

1. Recall that $X_n \to_p X$ means that X_n converges to X in probability.

(a) Show that if $X = a$ and $X_n = a_n$ almost surely for real numbers $a, a_1, a_2, \ldots$, then $X_n \to_p X$ if and only if $a_n \to a$.

(b) Show that if $Z_n \to_p 0$ and $Z'_n \to_p 0$, then $Z_n + Z'_n \to_p 0$. Use this to argue that if $X_n \to_p X$ and $Y_n \to_p Y$, then $X_n + Y_n \to_p X + Y$.

2. Show that if $X_n \to_p X$ and $Y_n \to_p Y$, then $X_n Y_n \to_p XY$.

3. Given a probability space $(\Omega, \mathcal{F}, P)$, consider the space of real-valued random variables modulo a.s. equivalence. Define a metric on this space by $d(X, Y) = E[\min\{1, |X - Y|\}]$. Prove that $X_n \to_p X$ if and only if $d(X_n, X) \to 0$; this shows that convergence in probability is metrizable. We will see later that almost sure convergence is (usually) not metrizable.

4. Durrett Exercise 2.2.1.

5. Durrett Exercise 2.2.6.