

Math 6710, Fall 2016
Homework 7

1. Prove the following limit theorems. *Hint:* Use Theorem 8.1 in the notes.

(a) (Upgraded Fatou) If $X_n \geq 0$ and $X_n \rightarrow_p X$, then $\liminf_{n \rightarrow \infty} E[X_n] \geq E[X]$.

(b) (Upgraded Dominated Convergence) Suppose that $X_n \rightarrow_p X$ and that there exists Y with $E[Y] < \infty$ and $|X_n| \leq Y$ a.s. for all n . Then $E[X_n] \rightarrow E[X]$.

2. Fix $\alpha > 0$ and let X_1, X_2, \dots be i.i.d. with $P(X_1 > x) = x^{-\alpha}$ for $x \geq 1$. Find $0 < \beta < \infty$ and a nondecreasing sequence of constants c_1, c_2, \dots such that

$$\limsup_{n \rightarrow \infty} \frac{\log X_n}{c_n} = \beta \quad \text{a.s.}$$

3. Suppose that X_1, X_2, \dots are independent with $P(X_n = 1) = p_n$ and $P(X_n = 0) = 1 - p_n$.

(a) Find and prove a necessary and sufficient condition in terms of the p_n for X_n to converge to 0 in probability.

(b) Find and prove a necessary and sufficient condition in terms of the p_n for X_n to converge to 0 almost surely.

4. Durrett Exercise 2.3.11.

5. Durrett Exercise 2.3.17.