Math 6620, Problem Set 2
Due Tuesday, February 19.

1. [L] 4-1.
2. [L] 5-2.
3. [L] 5-3.
4. We can define a *pseudo-Riemannian manifold* to be a smooth manifold M together with a smooth choice of non-degenerate inner products on the tangent spaces $g \in \mathcal{T}^2(M)$. (The key difference is that we do not require that g is positive definite.) Show that, given such a g, there is a unique connection ∇ which is symmetric (ie torsion-free) and compatible with g.
5. Let (M, g) be Riemannian, and let $d(\cdot, \cdot)$ be the corresponding distance function. Show that the topology induced by the distance function is the same one that M started with.