Mathematics 6310

Assignment 2, due September 12, 2011

Continue reviewing undergraduate group theory as needed, especially the theory of group actions (4.1 or the equivalent part of any other algebra book). Read the rest of 6.1. Then do:

- 4.1 (pp. 116–117): 7, 9
- 6.1 (pp. 198–201): 12, 13, 18, 19, 24, 25

Additional problems:

1. (Parts (a) and (c) were used in a proof in class and only explained very quickly.) Let G be a finite group, N a normal subgroup, and p a prime number.
   (a) If P is a Sylow p-subgroup of G, show that $N \cap P$ is a Sylow p-subgroup of N.
   (b) Give an example to show that the normality assumption is necessary in (a).
   (c) If G/N is a p-group, show that $G = NP$.
2. Let G be a simple group of order 60. If $\pi = \{2, 5\}$ or $\{3, 5\}$, show that G does not have a Hall π-subgroup. [Presumably you could do this by direct computation, using the fact that $G \cong A_5$. Please don't use this fact. Suggestion: Use group actions.]