Mathematics 6310

Assignment 4, due September 26, 2011

Read about normalizers of Sylow intersections in 6.2. Read as much as you want about the simple group of order 168. Start reading 6.3. Then do:

- 6.2 (pp. 213–214): 13, 21
- 6.3 (pp. 220–221): 4, 12, 13

Additional problems:

1. In class I proved the existence of free objects in the category of groups in three steps: (1) State precisely the UMP the desired object should have; (2) prove that an object with that UMP exists; (3) give a concrete description of the elements of that object. It wasn't logically necessary to do steps (2) and (3) separately, but I did it because (2) can be done in a very easy, straightforward way, while (3) requires some trickery. In the present exercise I want you to construct free objects in other categories of algebraic objects, where "free" is defined exactly as in the category of groups. None of the examples require any trickery, so you can combine steps (2) and (3) if you want. In other words, your existence proof in (2) can simply directly exhibit the free object in a concrete way, making (3) redundant. Finally, you can confine yourself to the free object on two generators if it simplifies notation, but if you do this, at least convince yourself that you could carry out a similar construction with any set of generators. [Suggestion: If you don't quickly see the answer, first try to figure out what the free object on one generator looks like.] Here are the categories:
   (a) The category of vector spaces (over a given field).
   (b) The category of abelian groups.
   (c) The category of G-sets for a given group G.
   (d) The category of commutative rings with identity.
2. (a) Explain how the free group construction can be viewed as a functor $S \mapsto F(S)$ from the category of sets to the category of groups.
   (b) Show that this functor preserves sums. You will have to start by formulating precisely what it means to say that a functor preserves sums. Note: It is *not* obvious that sums exist in the category of groups, but you can do this exercise without knowing that. It all comes down to playing with UMPs.