Mathematics 6310

Assignment 6, due October 17, 2011

Read 7.6, 9.6 through Proposition 26, the first part of 15.1, and the first page of the handout on Zorn's lemma. [In 9.6 you may omit the proofs of Lemma 25 and Proposition 26.] Review the basic definitions about modules (first part of Chapter 10). Then do:

- 7.4 (pp. 256–260): 36
- Handout on Zorn (p. 3): 1, 2
- 7.6 (pp. 267–269): 7
- 9.6 (pp. 330–335): 10, 11, 14, 17
- 15.1 (pp. 668–673): 4, 11 [You can accept the result of 8; I'll probably prove it in class or assign it as homework later.]

Additional problems:

1. Recall that if k is a commutative ring, then a *k-algebra* is a ring R whose additive group has the structure of k-module, such that the multiplication map $R \times R \to R$ is k-bilinear. Assume that k is a field and that R is a commutative ring k-algebra that's finite dimensional as a vector space over k. Prove that every prime ideal in R is maximal.
2. Let M be a finitely generated module over an arbitrary ring. If N is a proper submodule of M, prove that N is contained in a maximal (proper) submodule.