Mathematics 6310

Assignment 8, due November 8, 2010

Read 9.3, 9.4, 13.1, 13.2, 13.4, the relevant parts of 13.5, and the handout on the primitive element theorem. Then do the following:

- 9.4 (pp. 311–313): 2(c) [Hint: I don't like the book's hint.]
- 13.2 (pp. 529–531): 19, 20, 21 [Note: These exercises indicate a method that you could have used in Exercise 7.3.14.]
- 13.4 (p. 545): 5
- 13.5 (pp. 551–552): 5, 7

Additional problems.

1. Deduce from the proof of the primitive element theorem that $\sqrt{2} + \sqrt{3}$ is a primitive element in the extension $\mathbb{Q}(\sqrt{2}, \sqrt{3})/\mathbb{Q}$.
2. By the *universal* $n \times n$ *matrix* I mean the matrix $X = (x_{ij})$ in the polynomial ring $R := \mathbb{Z}[x_{11}, ..., x_{nn}]$ in n^2 variables. Show that the determinant of X is irreducible in R. [This is intuitively clear, since if there were a universal factorization of the determinant, you would have learned about it in your first linear algebra course. For a rigorous proof, view $\det X$ as a polynomial in x_{11} with coefficients in the polynomial ring in the other variables, and apply Gauss's lemma. If this seems too trivial, you're probably forgetting to check something.]