Mathematics 6310

Assignment 10, due November 14, 2010

Read the parts of 15.1 that you haven't read yet, up to the section on computations (p. 664). [Read about computations too if you're interested, but we won't have time to cover this material in class.] Read 15.2 up to primary decomposition. Then do the following:

- 15.1 (pp. 668–673): 16, 19, 20, 25, 26, 28* [The asterisk means that you don't have to do this problem; but you should read the statement of it and then say to yourself, "Oh, so that's why people care about tensor products."]
- 15.2 (pp. 686–691): 21

Additional problem.

Recall that I discussed $\mathbb{A}^1 \smallsetminus \{0\}$ in class and suggested that it should be thought of as an "abstract affine algebraic set" with coordinate ring $k[t, t^{-1}]$. Here t is the coordinate function on $\mathbb{A}^1$. Generalize this example as follows. Let V be an affine algebraic set with coordinate ring $R := k[V]$. Let $f \in R$ be arbitrary, and consider the open subset

$$V_f := \{x \in V : f(x) \neq 0\}$$

as in Exercise 15.2.21. Let R_f be the ring of fractions of R obtained by inverting f. [If you haven't seen this before, look at Theorem 36 on p. 707, and take $D := \{f^n \mid n \geq 0\}$.] Show that R_f can be identified with a ring of functions on V_f and that (V_f, R_f) is an abstract affine algebraic set as defined in class. Thus you want to find an affine algebraic set W and a bijection $V \to W$ under which R_f corresponds to $k[W]$.