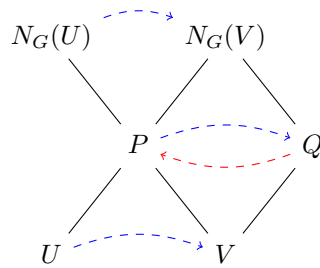


**Mathematics 6310**  
**Solution to Exercise 50 on p. 148**  
**Ken Brown, Cornell University, September 2011**

We are given a finite group  $G$ , a Sylow  $p$ -subgroup  $P$  of  $G$ , and two normal subsets  $U, V \subseteq P$ . If  $U$  and  $V$  are conjugate in  $G$ , we must show that they are conjugate in  $N_G(P)$ . [I think this result is called Burnside's fusion theorem, but I'm not positive.]

Choose  $g \in G$  so that  $gUg^{-1} = V$ . Then  $Q := gPg^{-1}$  is a Sylow  $p$ -subgroup of  $G$  containing  $V$ , and  $V$  is normal in  $Q$ . [Reason: Conjugation by  $g$  is an automorphism taking  $P$  to  $Q$  and  $U$  to  $V$ ; the assertion follows from the fact that  $U$  is normal in  $P$ .] So  $P$  and  $Q$  are both contained in  $N_G(V)$  and are therefore Sylow  $p$ -subgroups of the latter. By Sylow's theorem, there is an element  $h \in N_G(V)$  such that  $hQh^{-1} = P$ . Then  $hg$  normalizes  $P$  and conjugates  $U$  to  $V$ .

The following diagram summarizes the proof:



Here the dashed blue lines represent conjugation by  $g$ , and the dashed red line represents conjugation by  $h$ .