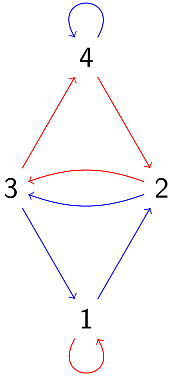
The Todd–Coxeter Procedure



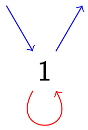
$$G := \langle\, a, b \,;\, a^3 = b^3 = (ab)^2 = 1 \,\rangle$$

$$H := \langle a \rangle$$

Let 1 be the coset H, so that

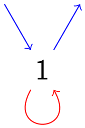
$$1^a = 1.$$

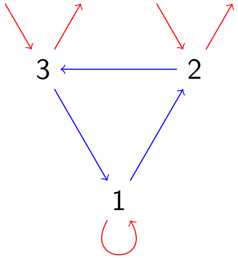
The Schreier graph of G acting on $H \backslash G$ (on the right) starts like this:



Check the relations at vertex 1.

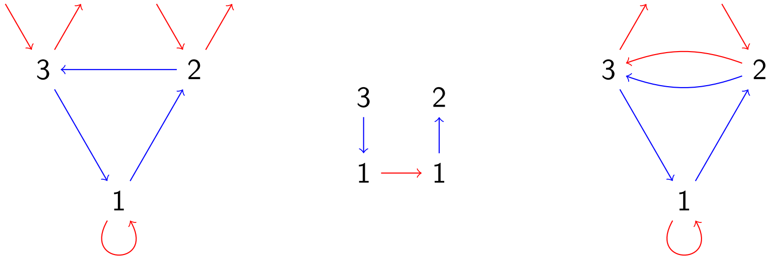
$a^3 = 1$ is OK. For $b^3 = 1$, define $2 := 1^b$ and $3 := 2^b$, deducing that $3^b = 1$.





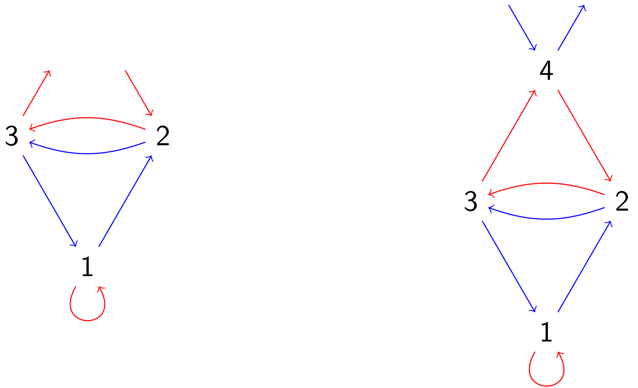
$abab = 1$?

Check the relation $abab = 1$ at vertex 1. We already have three sides of the desired square, so we deduce that $2^a = 3$:



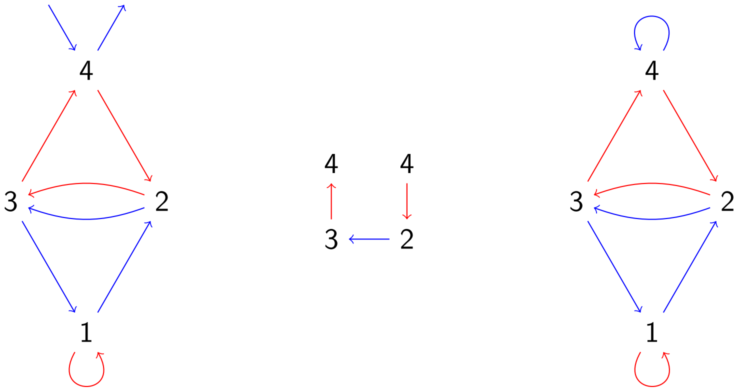
The graph is complete at vertex 1. Move on to vertex 2.

Check the relations at vertex 2. For $a^3 = 1$, define $4 := 3^a$ and deduce $4^a = 2$. The relations $b^3 = 1$ and $abab = 1$ already hold.



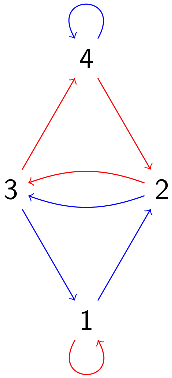
The graph is complete at vertex 2. Move on to vertex 3.

Check the relations at vertex 3. $a^3 = 1$ and $b^3 = 1$ are OK. For $abab = 1$ we again have three sides of the square, so we deduce that $4^b = 4$:



The graph is complete at vertex 3. Move on to vertex 4.

Check the relations at vertex 4. They're all OK.



The graph is complete at vertex 4. There are no more vertices. We're done.