1. Define \( \downarrow \) for sequences of sets and for sequences of real numbers, and verify that your definitions satisfy: If \((\Omega, \mathcal{F}, \mu)\) is a measure space and \(A, A_1, A_2, \ldots \in \mathcal{F}\) and \(A_n \downarrow A\) and \(\mu(A_1) < \infty\), then \(\mu(A_n) \downarrow \mu(A)\). Why is the condition \(\mu(A_1) < \infty\) needed?

2. Exercises 1.1.1–1.1.4, 1.2.4, 1.3.4, 1.4.1 in Durrett (Probability: Theory and Examples, 4th edition)

3. In a probability space \((\Omega, \mathcal{F}, P)\), prove that if \(A_1, A_2, \ldots \in \mathcal{F}\) are events such that \(P(A_n) = 1\) for all \(n \geq 1\) then

\[
P\left(\bigcap_{n \geq 1} A_n\right) = 1.
\]

Give an example of a family of events \(\mathcal{J}\) (necessarily uncountable) such that \(P(A) = 1\) for all \(A \in \mathcal{J}\) but

\[
P\left(\bigcap_{A \in \mathcal{J}} A\right) = 0.
\]

Due Thursday 9/11