

1. Define \downarrow for sequences of sets and for sequences of real numbers, and verify that your definitions satisfy: If $(\Omega, \mathcal{F}, \mu)$ is a measure space and $A, A_1, A_2, \dots \in \mathcal{F}$ and $A_n \downarrow A$ and $\mu(A_1) < \infty$, then $\mu(A_n) \downarrow \mu(A)$. Why is the condition $\mu(A_1) < \infty$ needed?
2. Exercises 1.1.1–1.1.4, 1.2.4, 1.3.4, 1.4.1 in Durrett (*Probability: Theory and Examples*, 4th edition)
3. In a probability space (Ω, \mathcal{F}, P) , prove that if $A_1, A_2, \dots \in \mathcal{F}$ are events such that $P(A_n) = 1$ for all $n \geq 1$ then

$$P\left(\bigcap_{n \geq 1} A_n\right) = 1.$$

Give an example of a family of events \mathcal{J} (necessarily uncountable) such that $P(A) = 1$ for all $A \in \mathcal{J}$ but

$$P\left(\bigcap_{A \in \mathcal{J}} A\right) = 0.$$