1. Show that the hypotheses of the monotone convergence theorem can be weakened to $f_n \uparrow f$ almost everywhere (a.e.) and $\int f_1^- \, d\mu < \infty$. Show by example that the theorem can fail if $\int f_1^- d\mu = \infty$.

2. Give an example of a random variable X such that $X(\omega) < \infty$ for all $\omega \in \Omega$ but $EX = \infty$.

3. Give an example of a random variable Y such that $Y(\omega) = \infty$ for some $\omega \in \Omega$ but $EY < \infty$.

4. Show that for a nonnegative random variable X on $(\Omega, \mathcal{F}, P)$,

$$EX = (P \times \lambda)\{(\omega, x) \,:\, 0 \leq x \leq X(\omega)\}$$

("the volume under the graph of X") and derive the familiar formula

$$EX = \int_{[0,\infty)} P(X \geq x) \, d\lambda(x).$$

In both of these formulas λ denotes Lebesgue measure.

5. An infinite family of collections $\{\mathcal{A}_i\}_{i \in I}$ is defined to be independent if and only if every finite subfamily is independent. Show that if $\{\mathcal{A}_n\}_{n \geq 1}$ is independent and $A_n \in \mathcal{A}_n$ for all $n \geq 1$ then

$$P(\bigcap_{n=1}^{\infty} A_n) = \prod_{n=1}^{\infty} P(A_n).$$

6. Exercises 1.5.6, 1.6.11, 2.1.1, 2.1.2, 2.1.18 in Durrett (*Probability: Theory and Examples*, 4th edition)

Due Thursday 9/25