1. **Sampling** from a probability measure μ means generating a random variable X whose distribution is μ. Suppose your only source of randomness is a fair coin, which you may flip as many times as you like (even a countably infinite number of times). That is, you can sample independent Bernoulli(1/2) random variables $X_1, X_2, \ldots$. Explain how to use the X_i to sample from the following distributions:

   (a) Unif$(0,1)$

   (b) Exp(1)

   (c) Bernoulli(p) for $0 < p < 1$

   (d) Unif(S_n) (the uniform distribution on permutations of $\{1, \ldots, n\}$)

   OPTIONAL: To avoid getting the impression that sampling is always easy, you might enjoy thinking about how to sample from

   (e) The uniform distribution on the n-dimensional unit ball $B = \{x \in \mathbb{R}^n : \sum x_i^2 < 1\}$.

   (f) The uniform distribution on spanning trees of a given finite graph G.

   (g) The uniform distribution on acyclic orientations of G which have a unique sink.

   You do not need to hand in parts (e)-(g)!

2. Fix $b > 0$ and let $X_n = \prod_{i=1}^n U_i$ where $U_1, U_2, \ldots$ are i.i.d. Unif$(0, b)$ random variables.

   (a) Prove that $\frac{\log X_n}{n} \to c$ a.s. for some constant $c = c(b)$.

   (b) Prove that there is a $\beta > 0$ such that $X_n \to 0$ a.s. if $b < \beta$ and $X_n \to \infty$ a.s. if $b > \beta$. Find β.

   (c) Suppose $b = 2.5$ and you are offered a deal which returns X_{1000} dollars tomorrow for an initial investment of \$1000 today. What is your expected gain? Explain why you would or would not take the deal.

3. Let $S_n = X_1 + \ldots + X_n$ be simple random walk in $\mathbb{Z}$. In class we proved that $S_n/\sqrt{n}$ converges weakly to a $N(0,1)$ random variable. Prove that $S_n/\sqrt{n}$ does not converge in probability to any random variable.

4. (a) If $X \stackrel{d}{=} X'$ and $Y \stackrel{d}{=} Y'$ does it follow that $(X,Y) \stackrel{d}{=} (X', Y')$?

   (b) If $X_n \stackrel{d}{\to} X$ and $Y_n \stackrel{d}{\to} Y'$, does it follow that $X_n + Y_n \stackrel{d}{\to} X + Y$?

5. Exercises 3.2.1, 3.2.12, 3.2.13 (compare to part b of the previous problem!), 3.3.1 in Durrett.

Due Thursday 10/23