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The future of prediction

Math awareness public lecture, Cornell, April 29, 2016

Some principles of prediction

- "Never assume what you're looking at is a random sample." (Nate Silver?)
- Explore the boundaries of your confidence and doubt.
- Extrapolate, but not blindly.
- If different approaches yield the same answer, increase your confidence.
- Beware of long chains of reasoning (A implies B implies C implies D implies E ....)

The future of prediction

- "Prediction is hard, especially about the future." –Neils Bohr
- (Like most great quotes, this one has been attributed to many different people!)

http://www.larry.denenberg.com/predictions.html

Predict the future of prediction?

- That’s meta.

A meta-prediction:

Prediction is self-limiting: A world full of predicting agents is a world that’s hard to predict!

(Financial markets, Keynes’ beauty contest, blue-eyed islanders,…)

Blue-eyed islanders

- 100 perfectly rational islanders
- 50 have blue eyes, 50 have brown eyes.

- If anybody learns his own eye color he must draw an X on his forehead that midnight.

- A stranger arrives and says to the entire group of 100, "at least one of you has blue eyes".

What happens?

http://www.xkcd.com/blue_eyes.html

Prediction is computation

- The future of prediction is the future of computation

(P=?NP, Impagliazzo's five worlds, …)

http://cstheory.stackexchange.com/questions/1026/status-of-impagliazzos-worlds

- A computational benchmark: Digits of pi=3.14159….

- Will we ever know the 10^100th digit?

Will we ever know the nth digit of pi?

- When $n=10^{100}$?

(Googol: 1 followed by 100 zeros)

- When $n=10^{10^{\wedge}100}$?

(Googolplex: 1 followed by Googol zeros)

- James Propp: "...it is likely that no physical process of computation in our universe would ever enable us to determine" the $10^{10^{\wedge}100}$th digit of pi.

https://mathenchant.wordpress.com/2015/12/17/really-big-numbers/

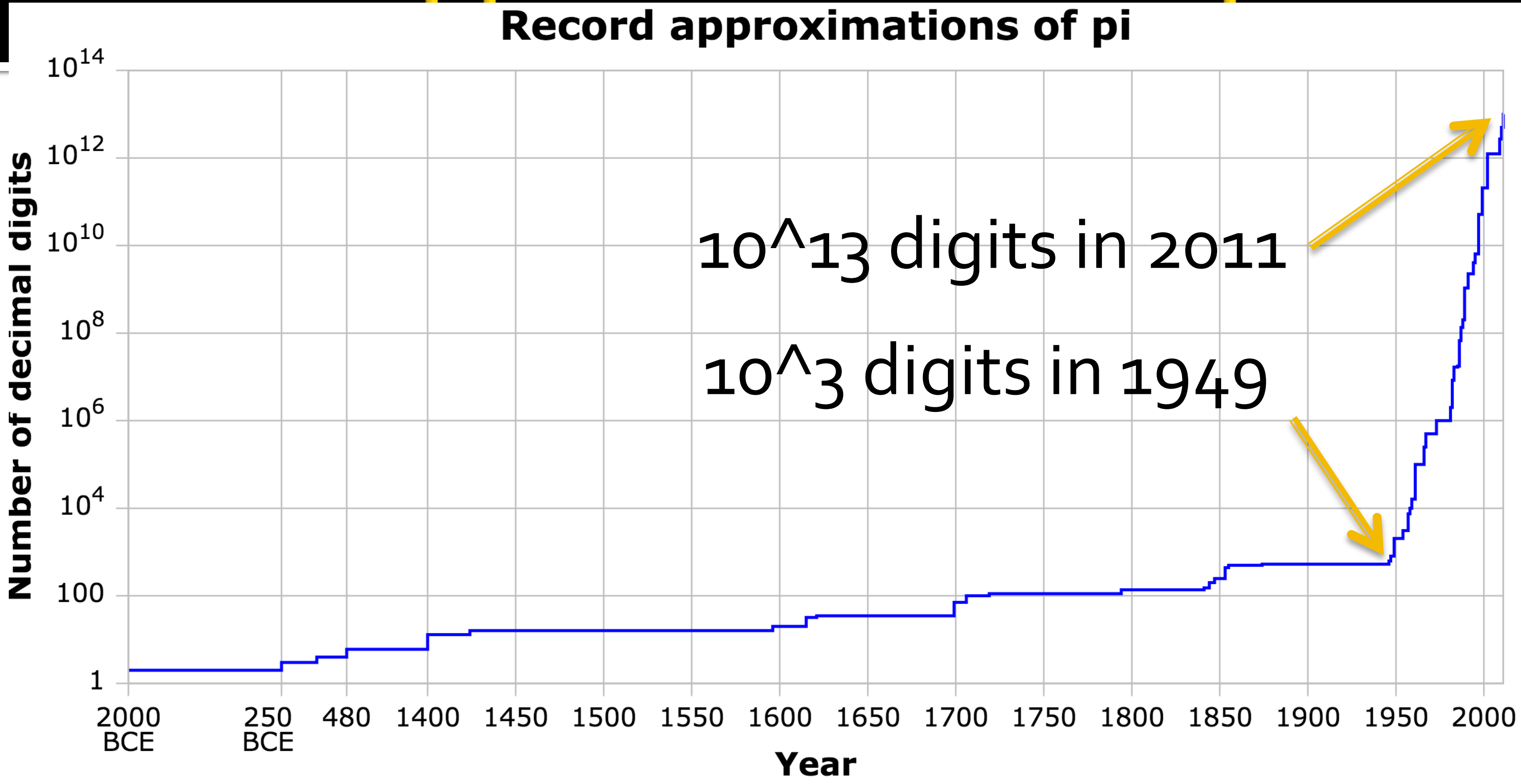
Them's fighting words!

- James Propp: "…it is likely that no physical process of computation in our universe would ever enable us to determine" the $10^{10^{100}}$th digit of pi.

- Me: Is it likely?

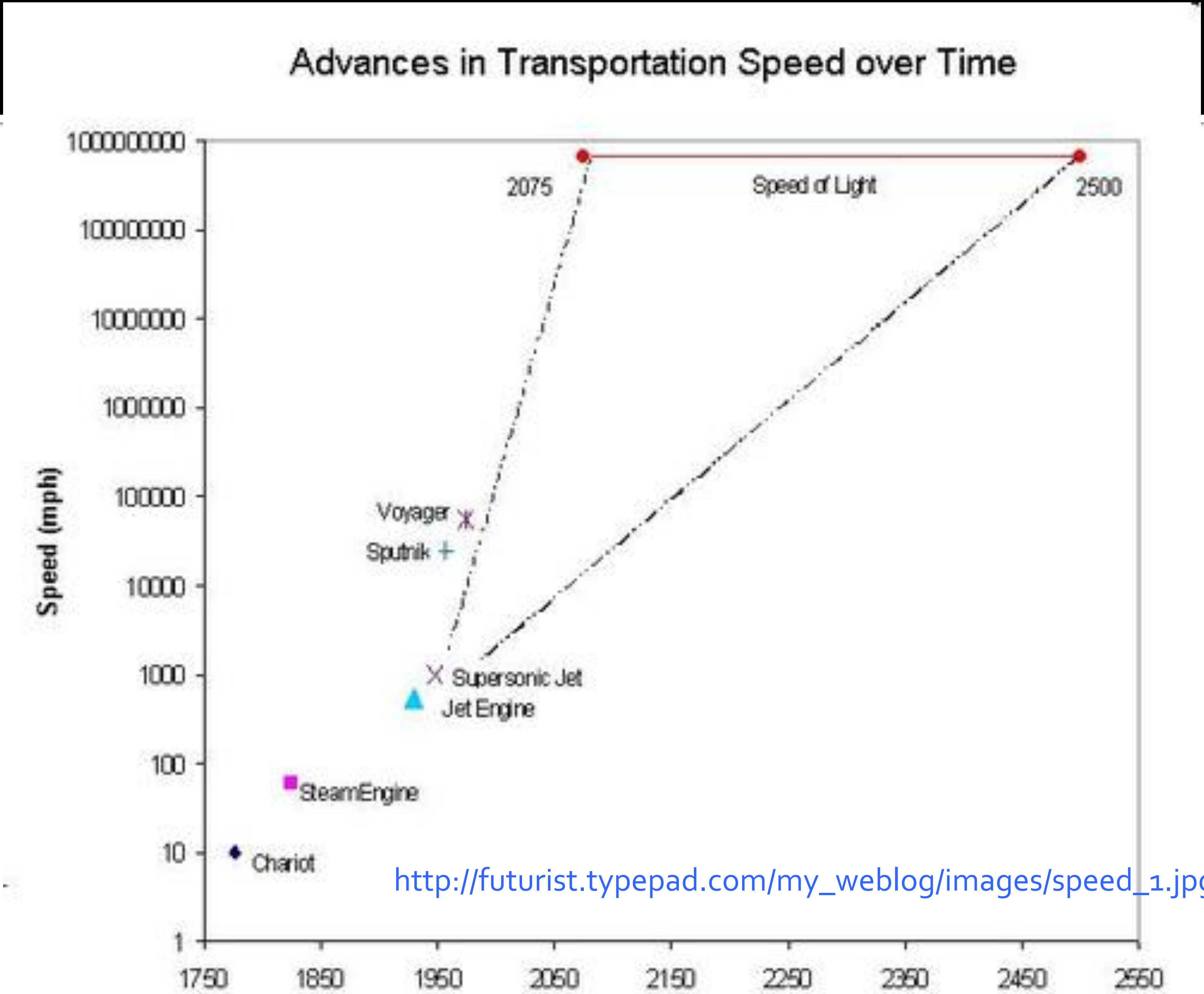
- JP: "That's me hedging my bets about whether the digits of pi contain some grand pattern that permits us to predict some of them. As hard as it is for me to imagine how such a thing could be true, it's even harder for me to imagine a way of proving that it's false!"

Record approximations of pi



Blind extrapolation: 10^100 digits in the year 2550?!

The dangers of blind extrapolation



http://futurist.typepad.com/my_weblog/images/speed_1.jpg

Is there a fundamental barrier?

- Storing 10^100 digits of pi could be impossible within the physical constraints of our universe (only 10^80 atoms!)
- At one digit per Planck time of 10^-51 years, a serial computation takes 10^49 years.
- What if we parallelize?

What if we parallelize?

- Rudy Rucker, 2009: “if we assume that we might master a eldritch quantum computational technique that lets us carry out one computational operation per [cubic] Planck length per Planck time, we’d be able to blaze along at 10^148 operations per second per cubic meter.

- It might actually be that our physical space is in fact doing this everywhere and everywhen…effortlessly. Just keeping itself going.

- Planet Earth has a volume in cubic meters of about 10^21, so if we throw all of the planet at a problem, we can compute 10^169 operations per second.”

Computing digits 1,…,n of pi

- Might not be hopeless for n=10^100.
- Seems completely hopeless for n=10^200.
- But what about computing just digit n?

- Is there a shortcut to the nth digit of pi that avoids computing digits 1,…,n-1?

- Indeed there is!

Bailey-Borwein-Plouffe formula

$$\pi = \sum_{k=0}^{\infty} \left[\frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \right]$$

Discovered in 1995 by Simon Plouffe.

Using modular arithmetic, it computes the nth digit of pi (in base 16) without computing digits $1, \ldots, n-1$!

Expect to be surprised!

- Questions a good forecaster should always be asking:
- “What’s the first thing that will surprise me?”
- “How surprised would I be to be proven wrong?”
- “What would convince me I’m wrong?”

An almost identity (Or, how to fool Wolfram Alpha!)

WolframAlpha computation knowledge en

(sum from k=-1000 to 1000 of e^(-(k/2)^2))

Sum:

Exact form

$$\sum_{k=-1000}^{1000} e^{-\left(\frac{k}{2}\right)^2} \approx 3.544907701811032105339319551261860174401$$

An almost identity (Or, how to fool Wolfram Alpha!)

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$$\sum_{k=-1000}^{1000} e^{-\left(\frac{k}{2}\right)^2} \approx 3.5449077018110321053393195512618601744 01$$

An almost identity (Or, how to fool Wolfram Alpha)

(sum from k=

Input:

$$2\sqrt{\pi}$$

Decimal approximation:

3.5449077018110320545963349666822903655950

Sum:

$$\sum_{k=-1000}^{1000} e^{-\left(\frac{k}{2}\right)^2} \approx 3.544907701811032105339319551261860174401$$

Hmmm...!

Whaaa...?

Fooled!

WolframAlpha computational knowledge engine

(sum from k=-10000 to 10000 of e^(-k^2/10000)) - 100*sqrt(pi)

Examples Rando

Input interpretation:

$$\sum_{k=-10000}^{10000} e^{-\frac{k^2}{10000}} - 100\sqrt{\pi}$$

Result:

0

Close but not quite!

(But what's an error $e^{-10000*pi\wedge 2}$ among friends?)

My first memory of a mathematical surprise

- 0,1,3,4,9,10,12,13,27,28,30,31,_____
- What comes next?

My first memory of a mathematical surprise

- 0,1,3,4,9,10,12,13,27,28,30,31,____
- 0,1,3,4,9,10,12,13,27,28,30,31,____
- Let's try writing in ternary (base 3).

My first memory of a mathematical surprise

- $1 = (1)_3$
- $3 = (10)_3$
- $4 = 3+1 = (11)_3$
- $9 = 3^2 = (100)_3$
- $10 = 3^2+1 = (101)_3$
- $12 = 3^2+3 = (110)_3$
- $13 = 3^2+3+1 = (111)_3$
- $27 = 3^2+3+1 = (1000)_3$

- nth term: write n in base 2, read in base 3.

The cheap solution

The On-Line Encyclopedia of Integer Sequences® (OEIS®)
https://oeis.org/ ▾ On-Line Encyclopedia of Integer Sequences ▾
Identify a sequence by entering few terms in the sequence. Additional information includes examples, formulas, and related links.
You visited this page on 4/19/16.

0, 1, 3, 4, 9, 10, 12, 13, 27, 28, 30, 31, ...

A greedy sequence

0, 1, 3, 4, 9, 10, 12, 13, 27, 28, 30, 31, ... Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A005836 Numbers n whose base 3 representation contains no 2.
(Formerly M2353)

0, 1, 3, 4, 9, 10, 12, 13, 27, 28, 30, 31, 36, 37, 39, 40, 81, 82, 84, 85, 90, 91, 93,
108, 109, 111, 112, 117, 118, 120, 121, 243, 244, 246, 247, 252, 253, 255, 256, 270, 2
273, 274, 279, 280, 282, 283, 324, 325, 327, 328, 333, 334, 336, 337, 351, 352 (list; graph
listen; history; text; internal format)

OFFSET 1,3

COMMENTS 3 does not divide binomial(2s, s) if and only if s is a member of this sequence, where binomial(2s, s) = A000984(s) are the central binomial coefficients.

This is the lexicographically earliest increasing sequence of nonnegative numbers that contains no arithmetic progression of length 3. - Robert Craigen (craigenr(AT)cc.umanitoba.ca), Jan 29 2001

In the notation of A185256 this is the Stanley Sequence S(0,1). - N. J. A Sloane, Mar 19 2010

Complement of A074940. - Reinhard Zumkeller, Mar 23 2003

Sums of distinct powers of 3. - Ralf Stephan, Apr 27 2003

Numbers n such that central trinomial coefficient A002426(n) == 1 (mod 3

Cool!

No 3-term arithmetic progressions

- A 3-term arithmetic progression (AP) is a sequence of the form $x, x+y, x+2y$.

Our sequence avoids them like the plague!

0,1,2 (oops, AP)
0,1,3,4,5 (AP)
0,1,3,4,6 (AP)
0,1,3,4,7 (AP)
0,1,3,4,8 (AP)
0,1,3,4,9,10,11 (AP)
0,1,3,4,9,10,12,13,27
[every number 14,…,26 would form an AP!]

A tempting conjecture

- If we greedily build a sequence containing no k-term arithmetic progression, the nth term will be "write n in base k-1, read the result in base k."

Seems right for k=3.

Let's try k=5:
0,1,2,3,5,6,7,8,10,11,12,13,15,16,17,18,25,…

Looking good.

In fact, it works whenever k is prime(!)

…but it's very false for k=4!

- Write n in base 3, read in base 4:
- 0,1,2,4,5,6 (is that a 4-term AP?!?)

Yep, here's another one (there are lots!)

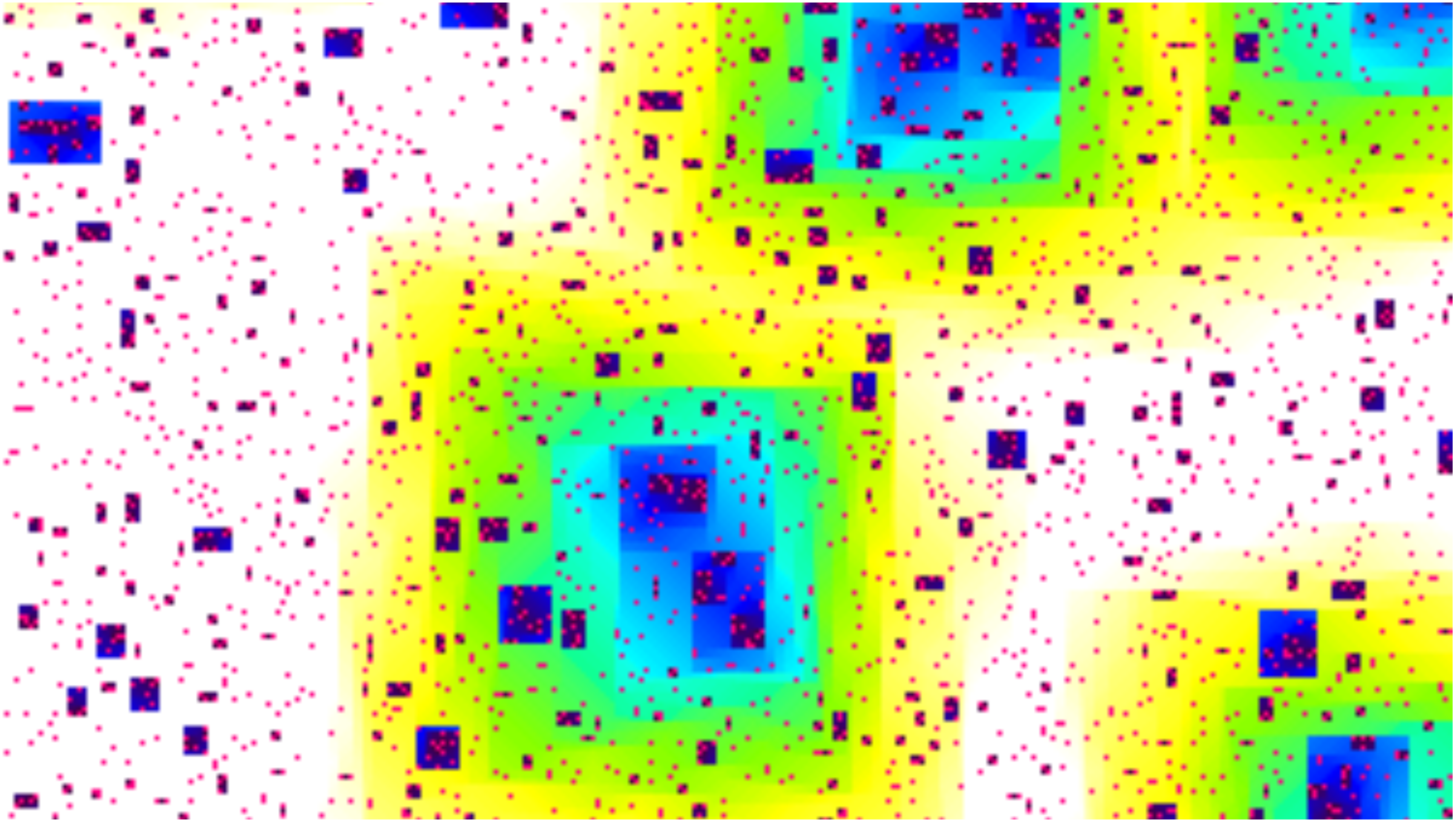
- 0,1,2,4,5,6,8,9,10,16,17,18,20,21,22,24,…

- Greedy no 4-term arithmetic progression:
- 0,1,2,4,5,7,8,9,14,15,16,18,25,26,28,29,…

Surprising earthworms

- Each square starts randomly with soil (probability p) or air (probability 1-p)
- Earthworm takes a random walk. She can push 1 square of soil but not 2 in a row!
- Experiment: It looks like the earthworm can avoid getting trapped if $p<0.4$.
- The truth: Even if $p=0.0000000000000001$ the earthworm eventually gets trapped!

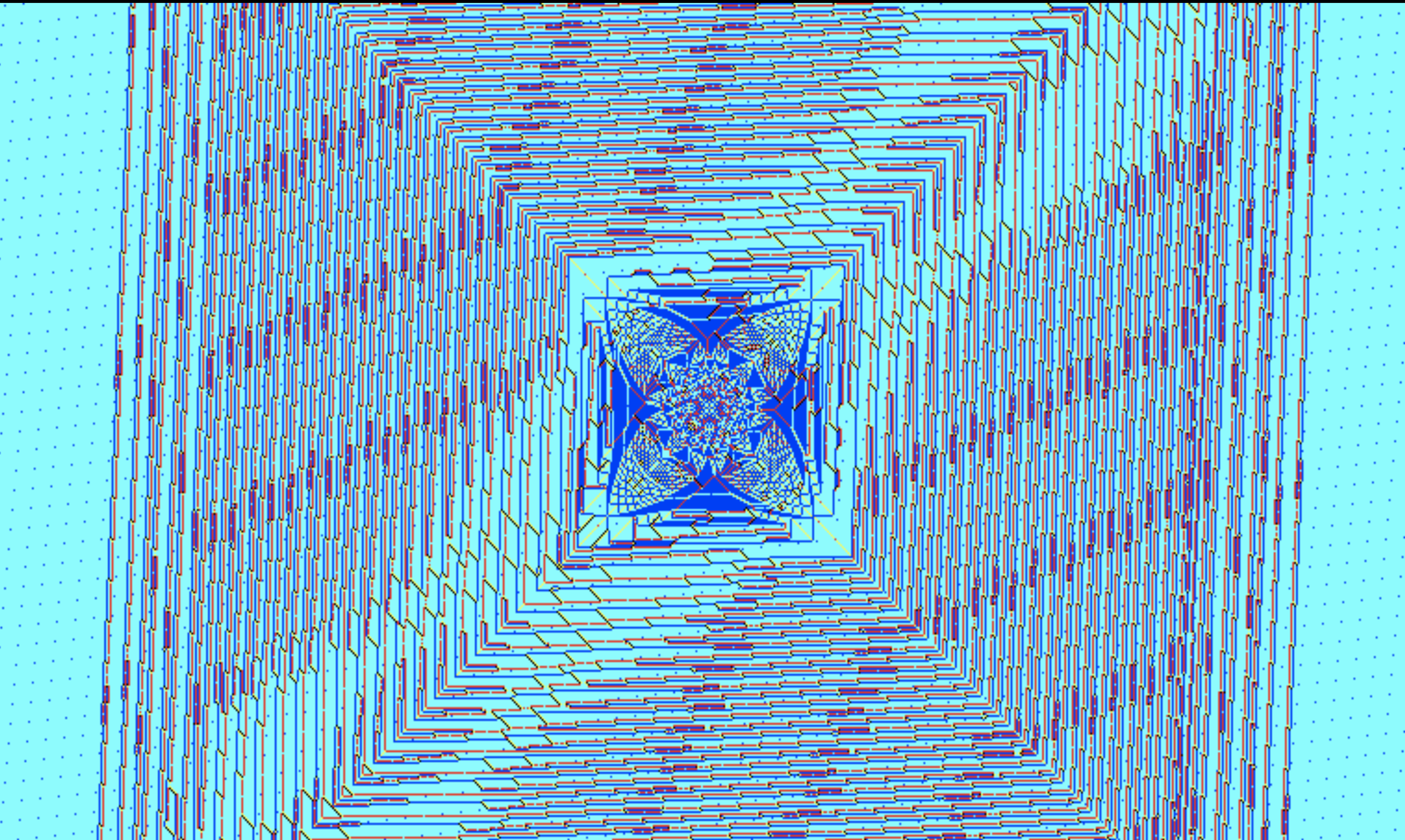
Surprising infections



Surprising infections

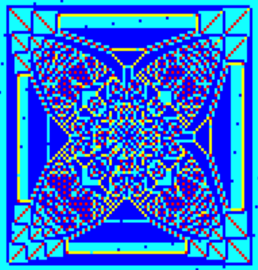
- Each square starts out “infected” with probability p.
- A square becomes infected if at least 2 neighboring squares are infected. Infected squares stay infected forever.
- Experiments say: If $p<0.01$ then most squares will never get infected.
- The truth: Even if $p=0.000000000000000001$, every square will eventually get infected!

Surprising sandpiles



Surprising sandpiles

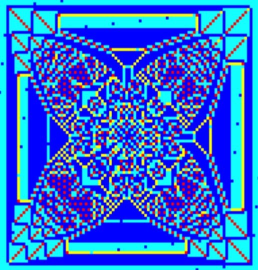
Drop 2257 grains of sand….



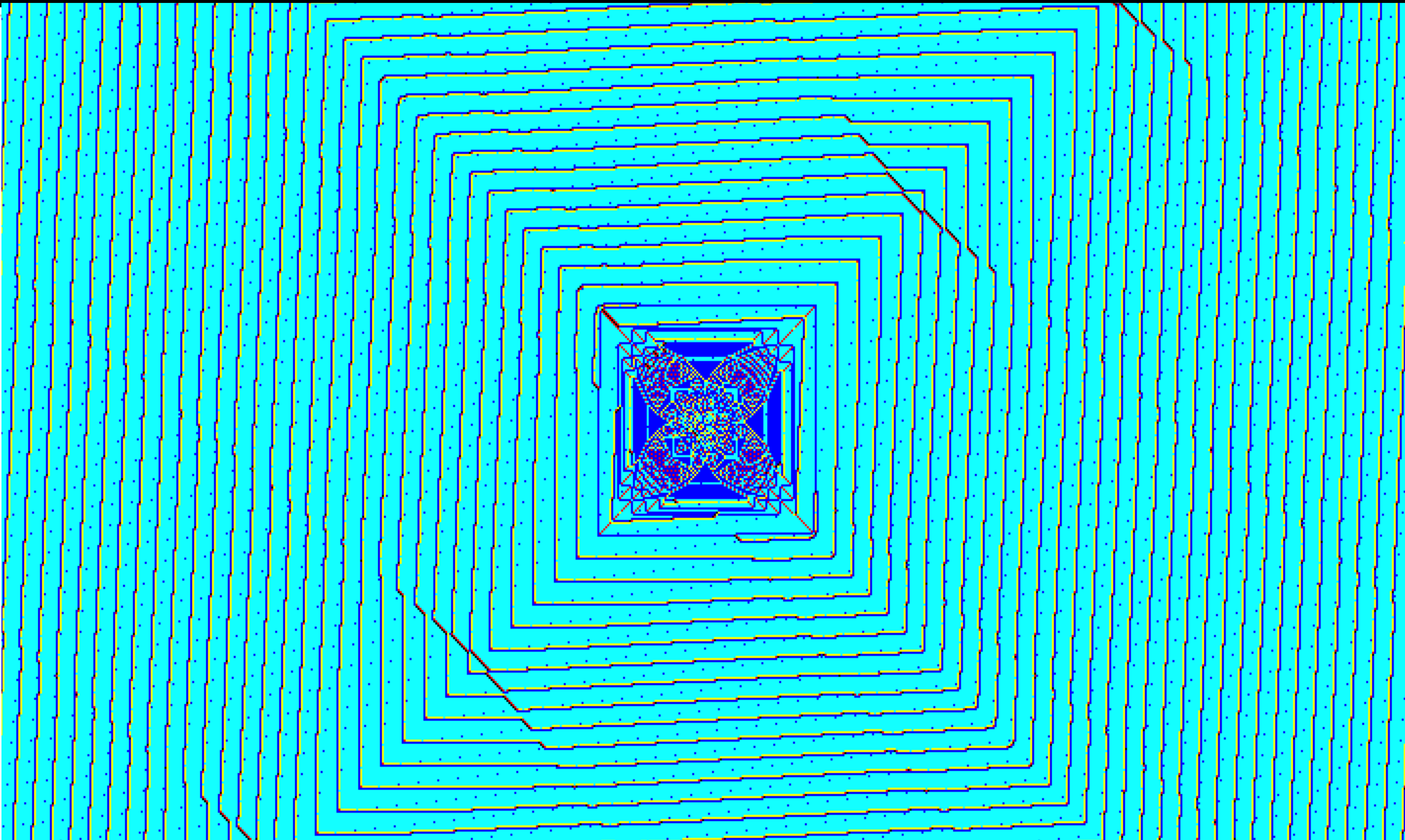
… and not much happens.

Surprising sandpiles

Drop one more grain of sand, for a total of 2258:



Boom!



Domino tilings: an anti-surprise!

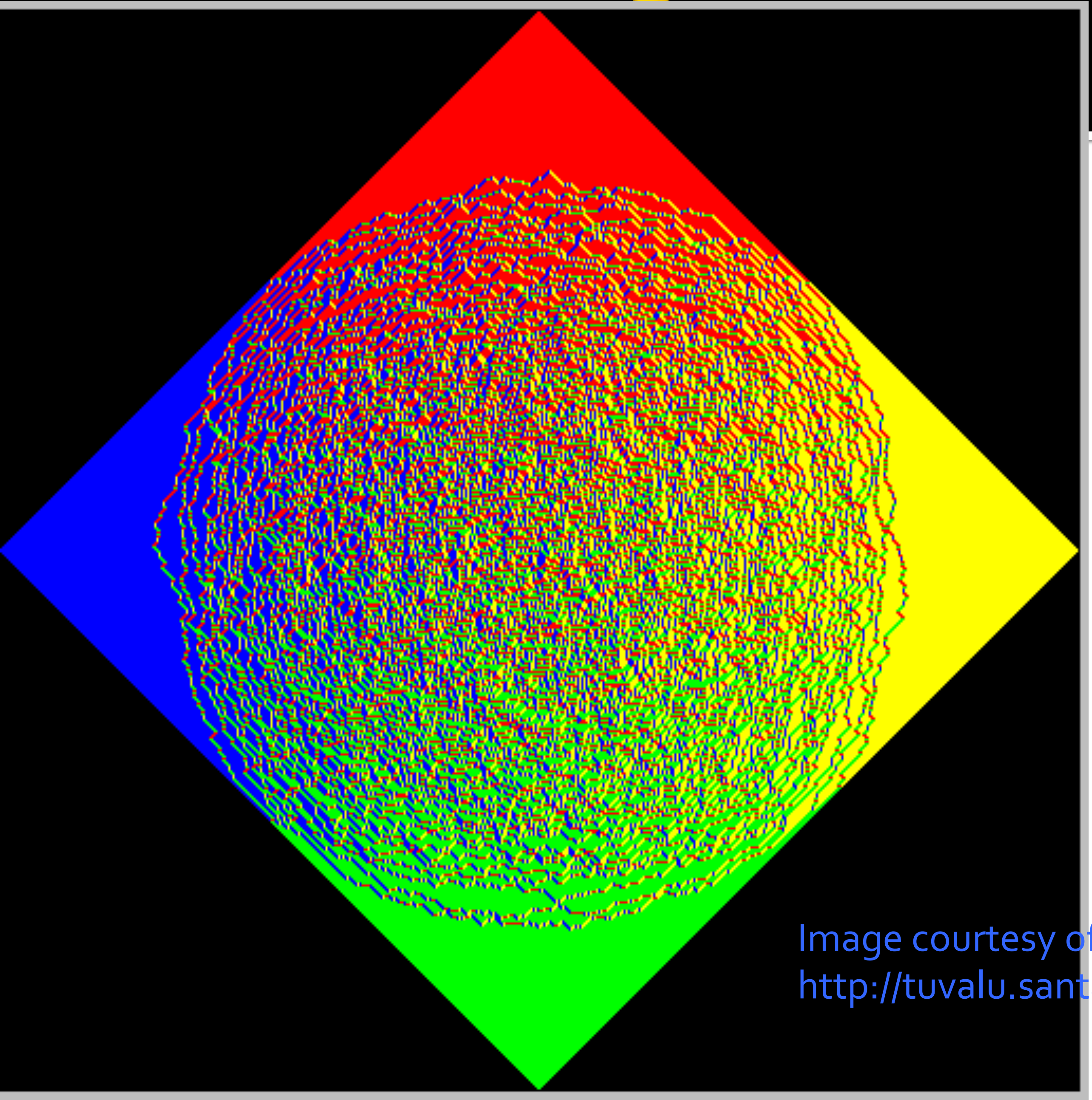
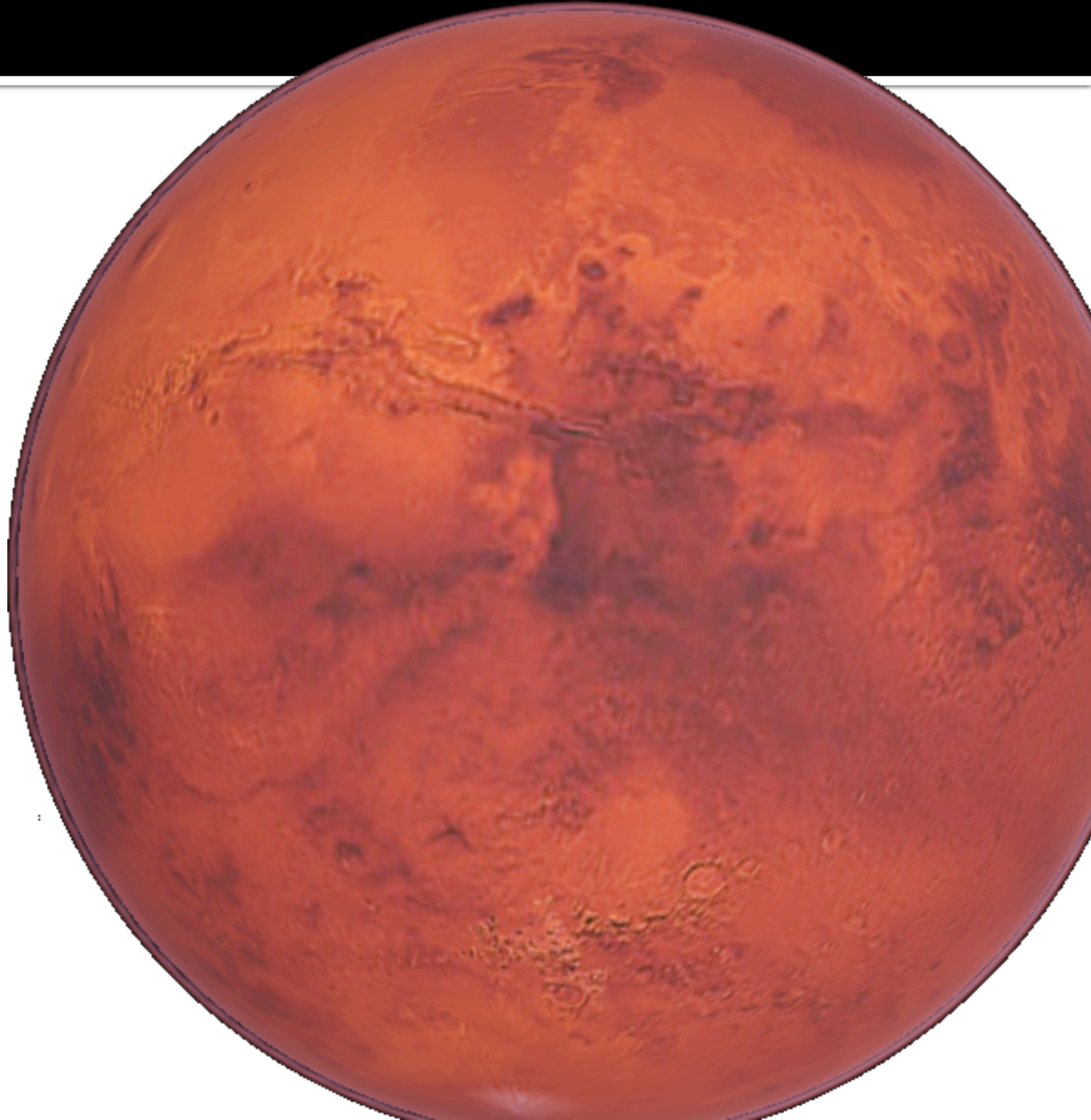


Image courtesy of Cris Moore
http://tuvalu.santafe.edu/~moore/aztec512.gif

Is there life on Mars?



…and should we hope the answer is No?

- "It would be great news to find that Mars is a completely sterile planet. Dead rocks and lifeless sands would lift my spirit"

  –Nick Bostrom, 2007

Why? Because of Hanson's "Great Filter"

The Great Filter

- *"Humanity seems to have a bright future, i.e., a non-trivial chance of expanding to fill the universe with lasting life. But the fact that space near us seems dead now tells us that any given piece of dead matter faces an astronomically low chance of begating such a future. There thus exists a great filter between death and expanding lasting life, and humanity faces the ominous question: how far along this filter are we?"* – Robin Hanson, 1998

Is the Great Filter in our past or our future?

Robin Hanson, 1998: "Consider our best-guess evolutionary path to an explosion which leads to visible colonization of most of the visible universe:

- The right star system (including organics)
- Reproductive something (e.g. RNA)
- Simple (prokaryotic) single-cell life
- Complex (archaeatic & eukaryotic) single-cell life
- Sexual reproduction
- Multi-cell life
- Tool-using animals with big brains
- Where we are now
- Colonization explosion

(This list of steps is not intended to be complete.) The Great Silence implies that one or more of these steps are *very* improbable"

Is the Great Filter still to come?

"Consider the implications of discovering that life had evolved independently on another planet in our solar system. That discovery would suggest that the emergence of life is not a very improbable event. If it happened independently *twice* here in our own back yard, it must have happened millions times across the galaxy. This would mean that the Great Filter is less likely to occur in the early life of planets and is therefore more likely still to come."
–Nick Bostrom, 2007

The good news

- Several plausible Great Filters in our evolutionary past:
  - Origin of Life
  - Eukaryotes
  - Sex
  - Multicellular Life
- And let’s not forget: SETI hasn’t found anyone!

The bad news, brought to you by SIA

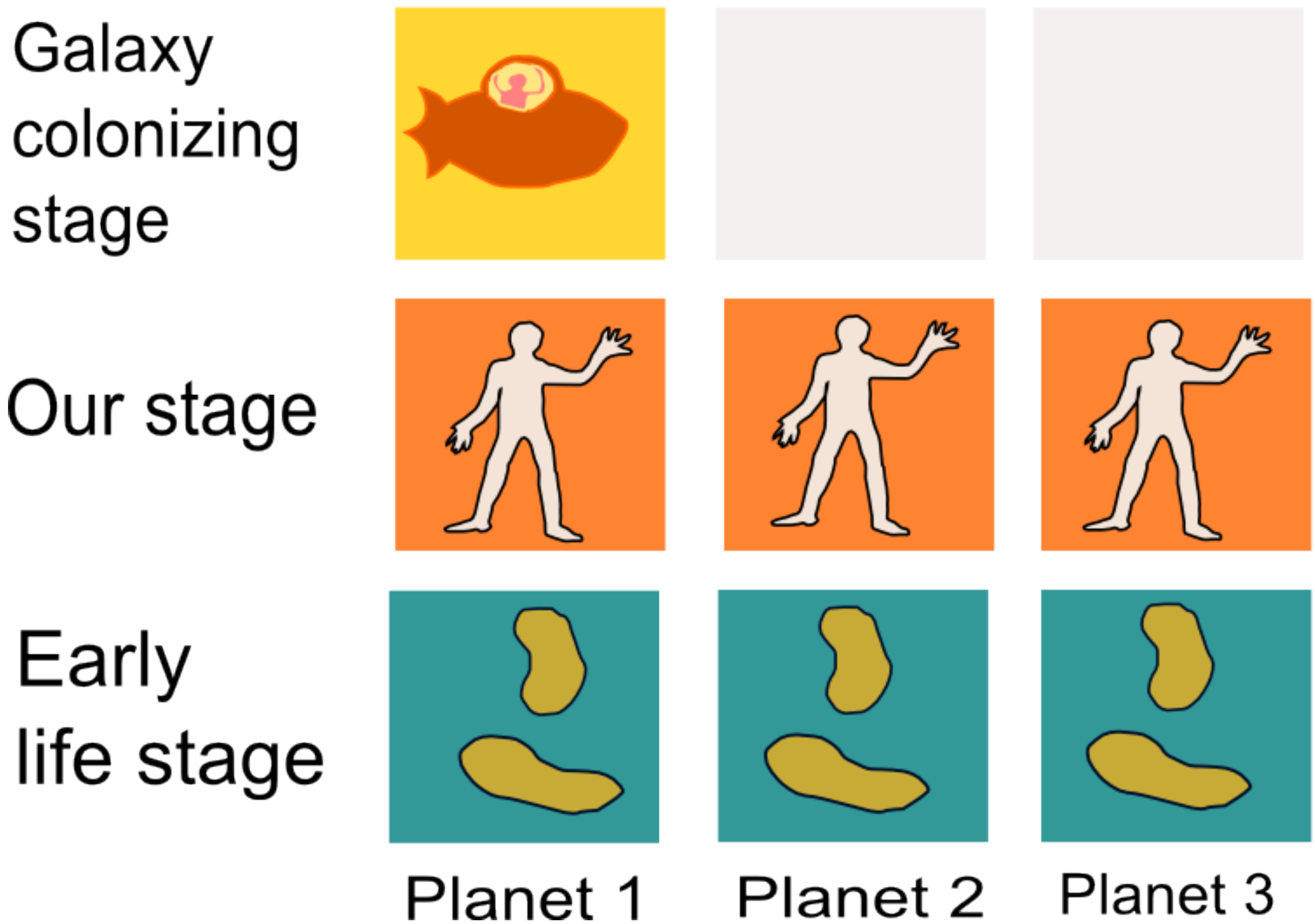
- Bostrom's Self-Indication Assumption (SIA):

"*All other things equal, an observer should reason as if they are randomly selected from the set of all possible observers*."

- Katja Grace, 2010: SIA implies the Great Filter is probably in our future. (Uh oh!)

- https://meteuphoric.wordpress.com/2010/03/23/sia-doomsday-the-filter-is-ahead/
- http://www.overcomingbias.com/2010/03/very-bad-news.html

Katja Grace's argument, in pictures



Katja Grace's argument, in pictures

SIA predicts higher likelihood of World 3!

I hope I conveyed that

- Randomness can be tamer than you think.
- Determinism can be wilder than you think.
- Selection bias is unavoidable, but the wily forecaster can turn it to her advantage.

Finally, if you want to make good predictions, never stop expecting surprises!

Thank you for listening!

- Special thanks to: Jim Propp, Steve Strogatz, Good Judgment Project, National Science Foundation, Sloan Foundation

- Some more things you can ask me:

  Is chess a win for black?

  Is Peano arithmetic consistent?

  What did the Netflix challenge do wrong?