TEACHING STATEMENT

LIONEL LEVINE

If there is one thing I have learned in my experience teaching, it is the incremental nature of the learning process. The most effective way to teach something new is by building it up in increments from old concepts with which the students are already familiar. In many cases this will not be the most "efficient" or "elegant" approach to the subject, and as such it often goes against the natural instincts of the instructor. In teaching elementary linear algebra, I began with ambitions to emphasize the geometric meaning of linear transformations and associated concepts like kernel and image, rank, projection maps, and so on. I soon found, however, that my students were better equipped from past experience to understand things initially in terms of solving systems of linear equations, and in terms of explicit algorithms like row-reduction and least-squares. I concluded that a high-level conceptual understanding is something we can impart to our students only gradually. The best route to such an understanding is one that weaves through incremental extensions of what the students already know.

In order to effectively teach our students anything new, we must therefore have an idea of what they already understand. This underscores the crucial importance of getting into the mindset of the student, which is often a significant challenge. In first semester calculus, for example, I felt that while my students were reasonably adept at calculating limits, they were missing some basic intuition about rates of growth. I designed a worksheet which defined f as "growing faster" than g if $\lim_{x\to\infty} f(x)/g(x) = \infty$ and worked out a few examples, then gave them a list of simple functions like x^2, x^3, 2^x, e^x and asked them to sort the list in order of how fast the functions grow. The students were initially quite perplexed by this task, until they realized the problem could be reduced to a series of problems of the form "Find the limit of $f(x)/g(x)$ as $x \to \infty$," with which they were already very familiar. It was gratifying to watch my students proceed from an unfamiliar and scary-looking problem to one which they knew how to solve. Although this reduction seems trivial to mathematicians, the thought process involved in "unraveling definitions," thereby transforming an unfamiliar problem into a familiar one, was one my students found highly nontrivial. It was also valuable for them to discover how a problem like evaluating a limit might arise in some other way than magically off the page of a calculus text. For my calculus and linear algebra classes at Berkeley, I wrote a series of 44 worksheets of this type designed to place the course topics in a broader conceptual context. I used these worksheets during group work in the discussion section to get the students involved in thinking about and discussing the material in novel ways. More recently at MIT I have had the opportunity to teach some higher-level material as part of MIT's Mathematics Lecture Series for undergraduates, and by giving several guest lectures in Scott Sheffield's graduate topics course in probability. I have also enjoyed serving as a one-on-one mentor for a number of students through MIT's Undergraduate Research Opportunities Program.

A challenge I plan focus on going forward is teaching mathematics in a way that also helps the students develop more general life skills. After all, the typical calculus student will not be taking derivatives on a daily basis after she graduates; what she *will* need are the skills to construct a logical argument backed up by data, and to pass judgement on arguments advanced by others. Many such arguments are constructed to be deliberately misleading, and others suffer from common fallacies. Accordingly, I wonder whether we do our students a disservice by carefully sifting our lectures for mistakes and presenting only the clear, proven truth. Students need to see what a bad argument looks like. They need to see what a mistake looks like. Students who are on the alert for deceptions and mistakes will be more active participants in the classroom. They will learn the material better, and they will learn the art of exposing a spurious argument. When I teach algebraic combinatorics this spring, I plan to try out a feature called "find the mistake:" I will tell the students that each lecture contains a deliberate mistake, and encourage them to find it and point it out in class. I will choose common mistakes that mathematicians know to look out for, like implicitly assuming that a set is nonempty, or forgetting that there are two square roots. If nobody finds the mistake, I will point it out myself at the end of class.

Most students, I suspect, are motivated by the short-term reward of positive feedback: they want to "get the right answer" and they want the professor to confirm that it's right. Without minimizing the importance of positive feedback, I try to convey in my teaching something of the value of mathematics beyond mere problem-solving: The world is a fascinating and mysterious place, and mathematics is one of the best tools we have for understanding it. I try to encourage an exploratory mindset in my students, both because I feel that learning this way is more exciting, and because it leads to a deeper and more lasting understanding of the subject. In both research and teaching I am guided by the same basic principle: The ultimate reward for understanding something is having the opportunity to lead others to the same understanding.