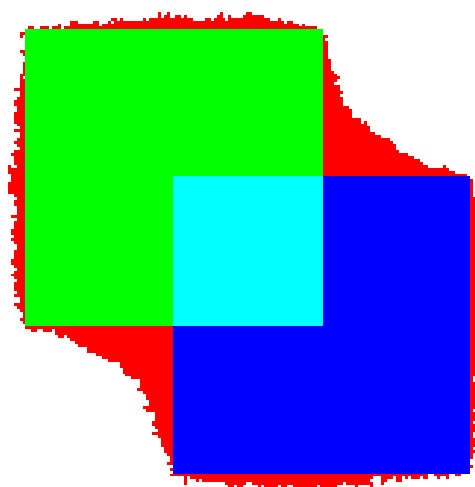


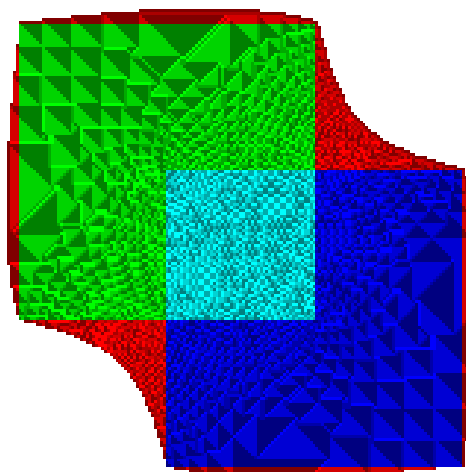
Free Boundary Problems Arising from Combinatorial and Probabilistic Growth Models

Lionel Levine

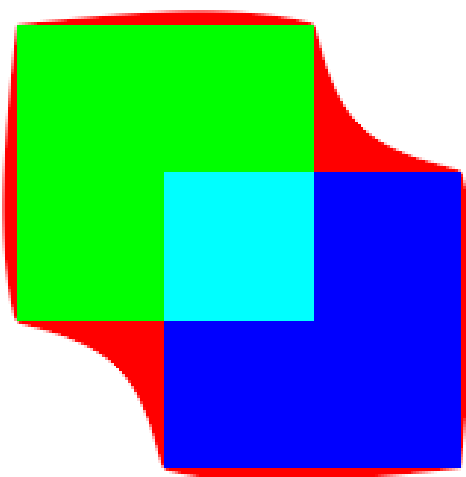
October 19, 2007



Internal DLA



Rotor-Router Model

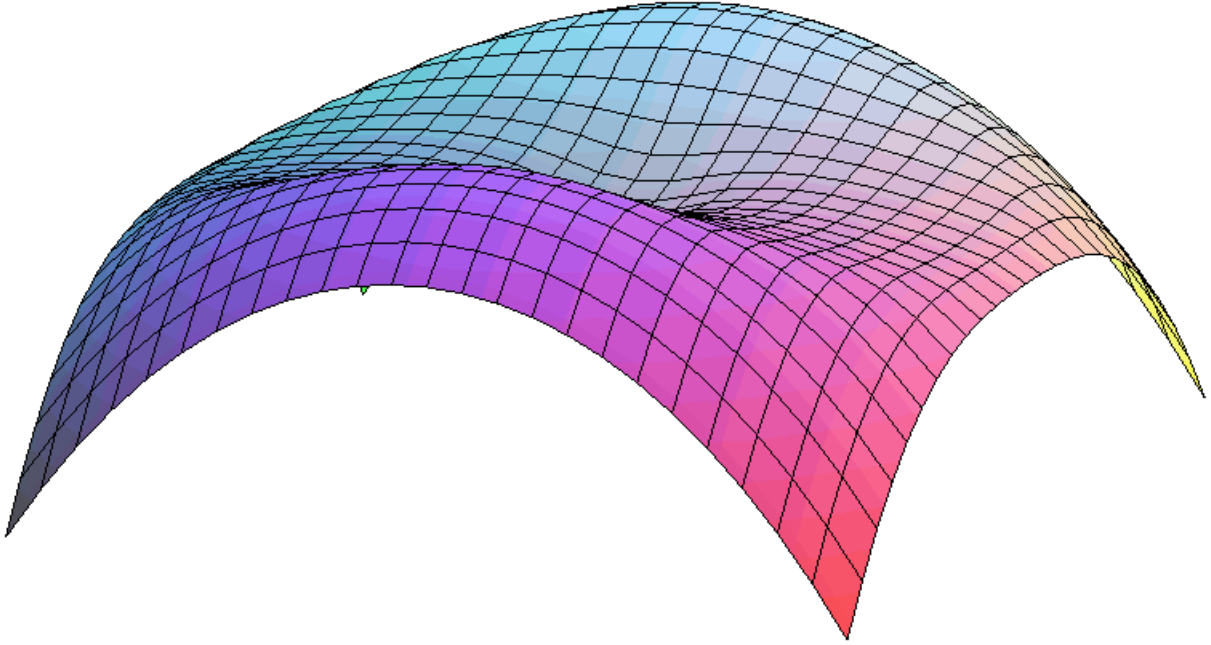


Divisible Sandpile

Figure 1: Smash sum of two squares in \mathbb{Z}^2 intersecting in a smaller square.

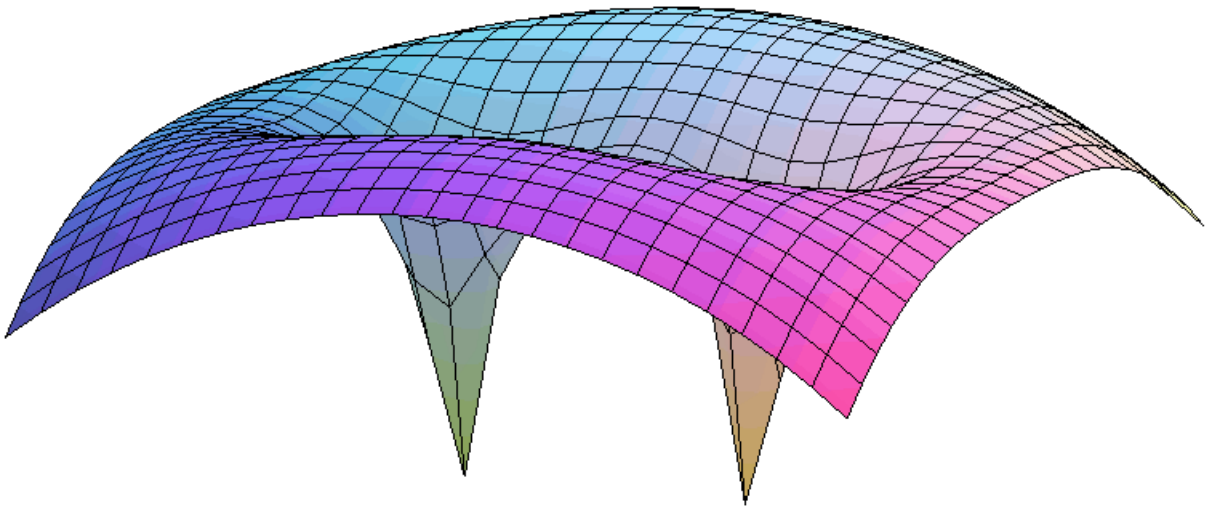
The obstacle for two overlapping disks A and B :

$$\gamma(x) = -|x|^2 - \int_A g(x,y)dy - \int_B g(x,y)dy.$$

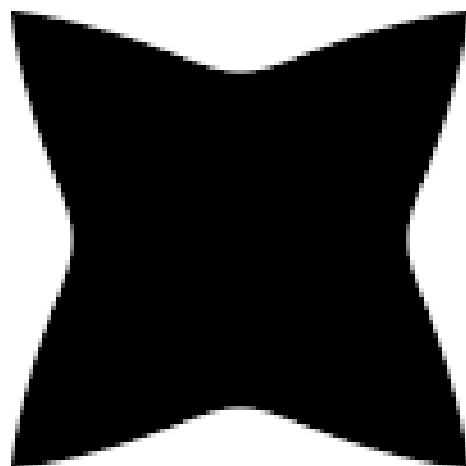


The obstacle for two point sources x_0 and x_1 :

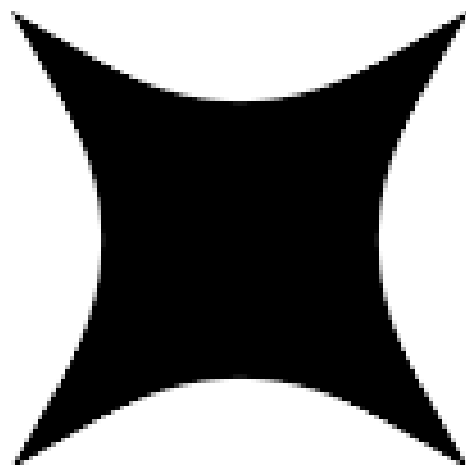
$$\gamma(x) = -|x|^2 - g(x,x_0) - g(x,x_1).$$



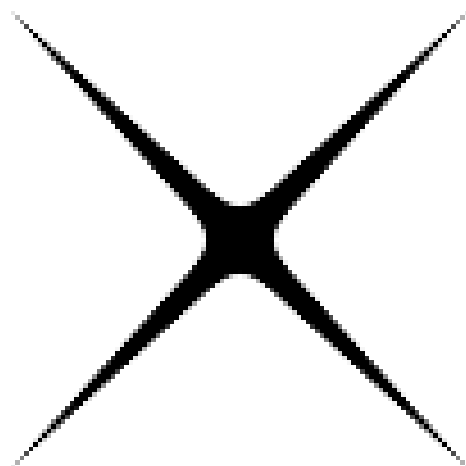
A domain C such that $B \oplus C$ is the unit square, where B is a disk of area $\frac{1}{4}$ centered at the origin:



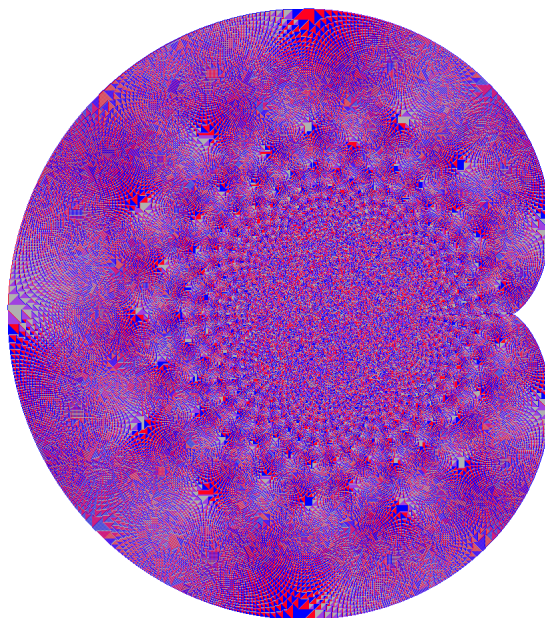
A domain D such that $D \oplus D$ is the unit square:



A domain E such that $E \oplus \dots \oplus E$ (10 summands) is the unit square:



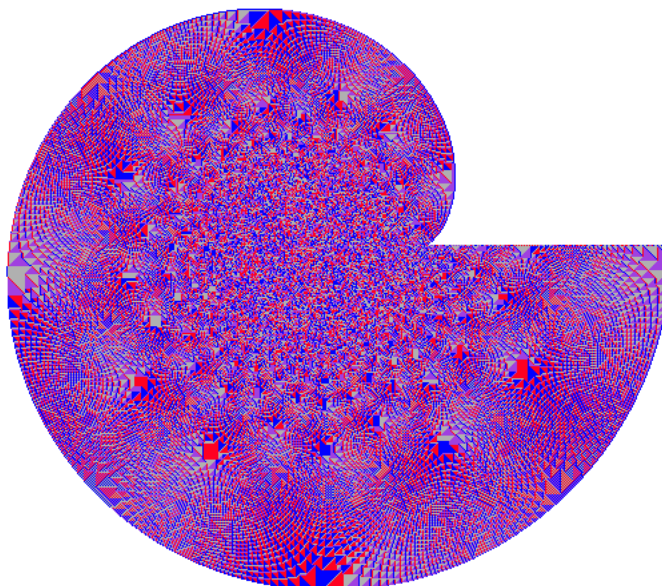
Rotor-router model with point source of particles, if rotors on the positive x -axis reflect particles back in the direction they came from:



Conjecture: this is the cardioid whose boundary is given by the curve

$$\pi^2|x|^4 - \pi|x|^2 + \frac{2\sqrt{2\pi}}{3\sqrt{3}}x_1 - \frac{1}{12} = 0.$$

Rotor-router model with point source of particles, if rotors on the positive x -axis always send particles in the downward direction:



What is this shape??