Ode to Geometric Group Theory

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Prologue: *Most students first learn of groups, as did the author, as abstract groups, axiomatically defined. In* geometric group theory *groups are viewed as a) certain sets of transformations or b) as metric spaces. The former view initiated group theory in the nineteenth century and has been an important theme in the field ever since. In the latter view, the elements of a group are viewed as vertices of a graph — Cayley's colour graph (1878), and Dehn's Gruppenbild (1910) — and the distance between two group elements is the length of the shortest path in the graph from one to the other. This view was made central by Gromov in the mid 1980's when the field of geometric group theory was brought into focus. Drawing from combinatorial group theory, low dimensional topology, Riemannian geometry and algorithmic questions in group theory, geometric group theory is currently in full blossom.*

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We look not askance on those days of innocence when we knew groups, unclad and axiomatic, as sets with a binary operation, from which mere logic and sometime cleverness would a fine and elegant fabric weave; nor begrudge we such simple joys to those yet in this pristine state.

But who of us — having once been conquered by the full-bodied richness of a well presented group, or having been swept freely away by its cocompact action by homeomorphisms or (gasp) isometries on a waiting space (with all its cohomology hanging spectrally in the balance) or having seen its boundary on a sleepless night or having followed its quasigeodesics to their very ends — who of us would return from this garden to that ascetic plane from which we came?

Rather we entreat the Uninitiate, come, come with us through the garden gate, that in unison we might tessellate and together of that awesome Tree† of Knowledge taste.

†It is as yet unknown whether this is an $\mathbf{R}$-tree or a Λ-tree for some other ordered group Λ.