Cribbage

Cribbage is a card game where players race to accumulate points by forming certain combinations of cards over a series of hands. Each hand consists of two types of scoring opportunities. During the 'play,' participants take turns playing cards and make combinations involving both their own cards and their opponents' cards. Once the play is complete there is the 'show,' where players make combinations involving their own cards and a single shared starter card. We describe below the standard version of Cribbage for two players; variations for three, four, and six player games also exist.

Rules

Cribbage is played with a standard deck of 52 cards. Each card is assigned a value for counting: aces count as ones, face cards count as 10, and all other cards count as their numerical value. When considering runs, all of the cards are considered in the standard order with aces low only.

The Deal

The dealer deals six cards to each of the players. Each player then discards two of her cards face-down into the center of the table. These four cards form the 'crib' and will be used by the dealer during the show at the end of the hand.

The Starter

After all players discard into the crib, the non-dealer cuts the deck. The dealer then turns over the top card and places it face-up on top of the discard pile. This card is called the 'starter' and will act as part of all players' hands during the show. If the starter is a Jack, the dealer *immediately* scores two points. This is called scoring for 'his heals.'

The Play

Starting with the non-dealer, the players alternate turns playing a single card face-up and stating the sum of the counting value of all cards played by both players. This total must not exceed 31. If a player has no cards left that will keep the total at 31 or lower then the player says "go" and her turn is skipped. The remaining player earns one point for the 'go' and must play cards so long as she is able to do so. The play then continues with the count reset to zero and with the player who called 'go' playing first in the new sequence. If a player brings the total to exactly 31 then she earns a point (this is in addition to the point for the go, which will also always be earned in this situation).

Play continues in this manner until no players have any cards remaining in their hand. When a player runs out of cards it is treated as a go. In particular, the last person to play will always score at least one point.

During the play points are also scored for completing various combinations of cards. The scoring combinations are:

- 2 points for playing a card that brings the total to 15.

- 2 points for a pair - playing a card of the same *rank* as the previous card. Note that cards of different ranks do not count as a pair even if they have the same counting value (for example a jack and a queen are not a pair).
- 6 points for a triple - three consecutive cards of the same rank. This is also called a 'pair royal.'
- 12 points for a quadruple - four consecutive cards of the same rank. This is also called a 'double pair royal.'
- One point per card for straights of at least 3 cards. The straight does not need to be played in sequential order to be scored, but no non-straight cards can break up the straight. For example, 4,2,3 does count as a straight while 2,3,6,4 does not.

In all cases, when the count is reset, all combinations are reset. For example, starting a new sequence with a card of the same rank as the last card of the previous sequence does not count as a pair.

An example hand:

Non-dealer plays a three and says "three".

Dealer plays a three, says "six for two" and pegs two points for the pair.

Non-dealer plays a three, says "nine for six" and pegs six points for the triple.

Dealer plays a six, says "fifteen for two" and pegs two points for the fifteen.

Non-dealer plays a five and says "twenty."

Dealer plays a four, says "twenty-four for four" and pegs four points for the straight (3,6,5,4).

Non-dealer plays a six, says "thirty for three" and pegs three points for the straight (5,4,6).

Dealer has only a seven left in his hand and says "go". The non-dealer has no cards left and therefore can make no more plays and pegs one point for the go.

The Dealer plays his seven, says "seven" and scores one point for playing the last card. Note that the dealer does not score for the (5,4,6,7) straight since the seven is part of a new sequence.

The Show

After all cards have been played the players score the points in their hands, starting with the non-dealer, then the dealer, then the crib (which is scored by the dealer). Each player's hand consists of the four cards that she did not put into the crib as well as the starter card. In the show, the scoring combinations are:

- 2 points for each combination of cards that total 15.
- 2 points for each pair of cards of the same rank. Therefore, three of a kind counts for 6 points (since there are 3 pairs of cards) and four of a kind counts for 12 points (6 pairs of cards).
- One point per card in a run of at least three cards.
- One point for a Jack (in the player's hand, not the starter card) of the same suit as the starter card. This is called one for "his nobs."

- If all four of a player's cards are of the same suit he scores four points for a flush. If the starter card is also of this suit, the player scores five points. In the crib, a flush can only be scored if all the cards are of the same suit as the starter card (in which case it is scored as five points).

Each card may be scored in multiple different combinations. For example, a hand of 10,5,5,4 with a starter card of 6 would count for 16 points: two points for the pair of 5's, three points for each of the two runs 4,5,6, two points for each of the fifteens with a 10 and a 5, and two points for each of the fifteens with a 4,5, and 6.

Winning

The first player to score 121 points wins the game. The game ends as soon a player reaches 121 points; this is why the order that points are scored can be very important, especially the fact that the non-dealer scores his hand first during the show. If the losing player fails to reach 91 points he is said to be "skunked." When playing a multi-game match, a skunking counts as two games. A player who fails to reach 61 points is said to be "double skunked."

The Cribbage Board

A cribbage board is usually used to assist with keeping score. While their are a variety of different boards available, the most standard versions today have two or three parallel tracks of 120 holes. Each player has two pegs which are used to mark his score, leapfrogging the pegs each time a new score is marked (this prevents a player from loosing track of his previous score). Some boards also have an area to mark games won by each player for multi-game matches.

Variation for Three Players

In the three player game, each player is dealt only five cards and one card is dealt directly into the crib. Each player then discards only one card into the crib (this leaves four cards in the crib and each player's hand). Play then continues as in the two player game, starting with the player to the left of the dealer and continuing clockwise.

Problems

All questions refer to the standard two person game. Note that until the play begins, the players have no knowledge of their opponent's hand. Therefore, from any single player's perspective, any card not in her hand has equal probability of being the starter card. For further discussion of this idea, see the problems section of the blackjack page.

1. You need eight points to win the match. After discarding an ace and a king into the crib you have 5,7,8,J left in your hand. What is the probability that your hand will score at least eight points in the show?

2. Since so many of the cards have a counting value of ten, fives are especially valuable cards. There are very few situations in which it is advantageous to discard a five into your opponents' crib. Consider

the situation where you have A,5,8,9,9,10 in your hand. If you discard the A,5 into the crib, what is the expected value for the number of cards of counting value 10 among the two remaining cards in the crib and the starter? In this problem, assume that the dealer's discard into the crib and the starter card are chosen at random from the cards not in your hand.

3. The highest score that can be earned during the show is 29 points. This occurs when you have three fives and a jack in your hand, with the jack a different suit from all of the fives, and the starter card is the remaining five. You earn twelve points for the quadruple, eight points for 15's involving the Jack, eight points for 15's involving just the fives, and one point for his nobs. What is the probability of obtaining this maximal hand?

4. You are dealt a hand of 2,2,3,6,7,9 with suits such that there is no chance for a flush. What is the expected value for your hand in the show if you keep 2,2,3,6? What if you keep 2,6,7,9?

Solutions

1. In this case, a 5,6,7,8,J, or the nine of the same suit as the jack in your hand will give you at least eight points in the show. So of the 46 cards remaining in the deck, a total of 17 will give the desired hand, giving a probability of (17/46)= 0.37.

2. There are a total of 46 cards not in your hand, so there are $C_{46,3} = 15,180$ different 3 card combinations for the starter and dealer's discard. Of the cards not in your hand, 15 have counting value 10. Therefore, the number of combinations with n cards of counting value 10 and $(3-n)$ cards of counting value less than 10 is $C_{15,n} \cdot C_{31,3-n}$. Therefore we have:

Value of n	0	1	2	3
Hands with n cards of value 10	4,495	6,975	3,255	455
Probability of n cards of value 10	0.296	0.459	0.214	0.030

Therefore, the expected value of the number of counting value 10 cards is

$$0 \cdot 0.296 + 1 \cdot 0.459 + 2 \cdot 0.214 + 3 \cdot 0.030 = 0.978$$

So on average we would expect approximately one card of counting value ten.

This is an example of the more general idea of a hypergeometric probability distribution. If you have an urn containing m red balls and n green balls and you choose N of these balls at random (without replacement), then the probability that you draw exactly k of the red balls is:

$$P(k) = \frac{(C_{m,k}) \cdot \left(C_{n,(N-k)}\right)}{C_{(m+n),N}}$$

The denominator gives the total number of ways of choosing N balls from the $n+m$ balls in the urn. The numerator is the product of the number of ways of choosing k of the m red balls and $N-k$ of the n green balls, which gives the total number of ways of chooing N balls with exactly k red.

The expected value of a general hypergeometric disribution is $N \cdot \frac{m}{n+m}$. This is the initial probability of drawing a red ball times the number of balls that will be drawn. In our example, we are picking three cards ($N = 3$), with 15 cards of couting value ten ($n = 15$) and 31 cards of counting value less than 10 ($m = 31$). So our expected value is $3 \cdot \frac{15}{15+31} = 0.978$.

3. First we determine the probability that our initial six card hand contains three fives and a jack, all of different suits, but not the remaining five (which needs to be the starter card). There are $C_{52,6} = 20,358,520$ different six card hands that we can be dealt. We can choose the three fives that we will be dealt in $C_{4,3} = 4$ different ways. Since the jack must be of a different suit than all of the fives, once we choose the three fives the jack is chosen as well. The final two cards in our hand can be any of the remaining cards except for the fourth five. These two cards can therefore be chosen in $C_{47,2} = 1,081$ ways. This gives a total of $4 \cdot 1,081 = 4,324$ different hands containing three fives and a jack all of different suits (but not the fourth five), giving a probability of $4,324/20,358,520 \approx 0.00021239$ of being dealt this type of hand.

In order to score a 29, we need the starter card to be the one five not in our initial hand. There are 46 cards not in our initial hand, so the probability of the last five being the starter is $1/46 \approx 0.02173913$. Therefore, the probability of getting a 29 is $0.00021239 \cdot 0.02173913 = 0.000004617$, or 1 in 216,580.

4. In the chart below we calculate the value of the show for each of the two hands depending on the rank of the starter card.

Hand	Ace	2	3	4	5	6	7	8	9	Ten	Jack	Queen	King
2,2,3,6	8	8	4	12	4	6	6	4	4	6	6	6	6
2,6,7,9	4	8	4	6	7	10	8	10	8	4	4	4	4

Multiplying each of these point values by the probability of the starter taking on each of the various ranks gives an expected value for the 2,2,3,6 hand of

$$\frac{4}{46} \cdot (8 + 12 + 4 + 4 + 6 + 6 + 6 + 6) + \frac{3}{46} \cdot (4 + 6 + 6 + 4) + \frac{2}{46} \cdot 8 = 6.17$$

and an expected value for the 2,6,7,9 hand of

$$\frac{4}{46} \cdot (4 + 6 + 7 + 10 + 4 + 4 + 4 + 4) + \frac{3}{46} \cdot (4 + 10 + 8 + 8) + \frac{2}{46} \cdot 8 = 6.04$$

So while the 2,2,3,6 hand only scores two points before the starter is included and the 2,6,7,9 hand scores 4 points, the 2,2,3,6 hand has a greater expected value once the starter is added. In an actual game, other factors such as the value of the cards being placed into the crib, the value of cards during the play, and the current score of the game, are combined with this type of expected value combination to determine the best four cards to keep.