Homework 10: Math 6710 Fall 2012

Due in class on Thursday, November 1.

1. Prove the conditional dominated convergence theorem: suppose $\mathcal{G}$ is a σ-field, X_n, X are random variables with $X_n \to X$ almost surely, and there is an integrable Z with $|X_n| \le Z$ a.s. Show that $E[X_n \mid \mathcal{G}] \to E[X \mid \mathcal{G}]$ almost surely and in L^1.

2. Let X, Y be iid integrable random variables. Compute $E[X \mid X+Y]$. (As in last week's homework, the answer will be $f(X+Y)$ for some measurable function f; try to find f explicitly. Hint: Think also about $E[Y \mid X+Y]$.)

3. (A very topical question) The Giants and Tigers are playing in the World Series. (This is a 7 game series and the first team to win 4 games wins the series.) Let ξ_n, $n = 1, \dots, 7$ be the winner of the nth game (G for Giants and T for Tigers), and let $\mathcal{F}_n = \sigma(\xi_1, \dots, \xi_n)$. Assume that the ξ_n are iid and that the teams are evenly matched,[1] so that $P(\xi_n = G) = P(\xi_n = T) = 1/2$.

   Suppose we bet \$1 on the Giants to win the series. Let M_n be the amount of money we have after the nth game, $n \le 7$. (Thus, $M_n = 1$ on the event that the Giants win 4 of the first n games, $M_n = -1$ if the Tigers do, and $M_n = 0$ if neither team has won 4 games yet, so the series is still going on.) Show that $E[M_n] = 0$ for all n, so this is a "fair" bet. Is $\{M_n\}$ a martingale with respect to the filtration $\{\mathcal{F}_n\}$?

4. Durrett Exercise 5.2.6.

5. Durrett Exercise 5.2.13.

[1] This is an unrealistic assumption. Actually, the Giants are clearly way better than the Tigers. Go Giants!