

# The Oliver Club

[www.math.cornell.edu/~oliver/](http://www.math.cornell.edu/~oliver/)

## ***The Space of Commuting Matrices and Statistical Mechanics***

*Why do matrices commute? More specifically, are there polynomial equations satisfied by the set of pairs of commuting matrices not algebraically implied by the equations  $AB = BA$ ? Mel Hochster and others asked this question in the '60s, and it remains unsolved for large dimensions. (To see the problem: knowing  $M^2 = 0$  tells you  $M$  is nilpotent and hence  $\text{Trace}(M) = 0$ , but that linear equation doesn't lie in the ideal generated by the quadratic equations  $M^2 = 0$ .)*

*I'll talk about some related spaces of matrices that are simpler to study, which lead to some weird integer-valued invariants of permutations. Then I'll explain a statistical mechanical model that produces the same integers, but in a much more calculable manner, and use this to give a formula for the volume (or really, multidegree) of the space of commuting matrices. This work is joint with Paul Zinn-Justin.*



## **Allen Knutson**

University of California, San Diego

Refreshments will be served at 3:55 PM in the  
Mathematics Department lounge (532 Malott Hall).

**Thursday, October 25, 2007  
at 4:25 PM in 406 Malott Hall**