

The Oliver Club

www.math.cornell.edu/~oliver/

Recent Progress on Sphere Packing

The problem of densely packing non-overlapping congruent spheres in Euclidean space of various dimensions N has a long history and rich connections with various aspects of modern pure and applied mathematics. For example, the Leech lattice yields a remarkably dense and symmetrical packing in dimension $N = 24$, conjectured to be the densest possible in this dimension. In this packing, each sphere touches 196560 others. It has been known for some time that this "kissing configuration" is optimal and unique. We review the proof of this and related facts and explain why the method cannot be used directly to study the sphere-packing problem. We then outline recent work on the sphere-packing problem, culminating in the proof by H. Cohn and A. Kumar that the conjectured optimality of the Leech packing holds under the condition that the centers of the spheres lie on a lattice (previously the densest lattice packing was known only for $N = 1$ through $N = 8$). Without the lattice condition, they show that any packing cannot improve on the Leech density by more than 1 part in 10^{30} . They also give strong evidence for the existence of mysterious functions of a positive real variable that would prove the optimality of the Gosset (E_8) and Leech packings of spheres for $N = 8$ and $N = 24$.

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Refreshments will be served at 3:55 PM in the Mathematics Department lounge (532 Malott Hall).

Thursday, March 27, 2008
at 4:25 PM in 406 Malott Hall

