A graph can be viewed, in many respects, as a discrete analogue of an algebraic curve. We begin by formulating the theory of “divisors” on graphs and on tropical curves, and discuss the combinatorial analogues of the classical Riemann-Roch theorem due to Baker-Norine, Gathmann-Kerber, and Mikhalkin-Zharkov. We then describe the analogues of Jacobian varieties in these settings, which are intimately related to the classical matrix-tree theorem. Connections with commutative algebra, algebraic geometry, and non-archimedean geometry will also be mentioned.

(The talk will be aimed at a general mathematical audience. The non-expository part of the talk will be based on joint works with M. Baker -- M. Baker, G. Kuperberg, A. Yang -- F. Mohammadi.)