Unitary representations of Lie groups appear everywhere in mathematics: in harmonic analysis (as generalizations of the sines and cosines appearing in classical Fourier analysis); in number theory (as spaces of modular and automorphic forms); in quantum mechanics (as “quantizations” of classical mechanical systems); and in many other places. They have been the subject of intense study for decades, but their classification has only recently emerged in a preprint authored jointly with Adams, van Leeuwen, and Vogan. Perhaps surprisingly, the classification has inspired connections with interesting geometric objects (equivariant mixed Hodge modules on flag varieties).

The purpose of this talk is to review the history and motivation behind the study of unitary representations, offer a few hints about algebraic and geometric approaches to them, and explain the key new idea which makes their classification possible. In retrospect, the new idea has the famous “unitary trick” of Hermann Weyl as its antecedent.