The Big Dehn Surgery Graph

We will describe a construction which allows one to go from one closed 3-manifold to another, called Dehn surgery, then use this construction to build an infinite graph whose vertices correspond to 3-manifolds. There is an edge between two vertices $v_M$ and $v_N$ if there exists a Dehn surgery along a knot in $M$ yielding $N$. We call this graph "The Big Dehn Surgery Graph". To our knowledge, it was first considered by W. Thurston. We metrize this graph by declaring each edge to be length 1. Questions about the combinatorics and geometry of the big Dehn surgery graph often correspond to interesting new and old questions about the structure of 3-manifolds. We present results about the link of $S^3$, the global geometry of this graph, and some interesting subgraphs. This is joint work with Neil Hoffman.