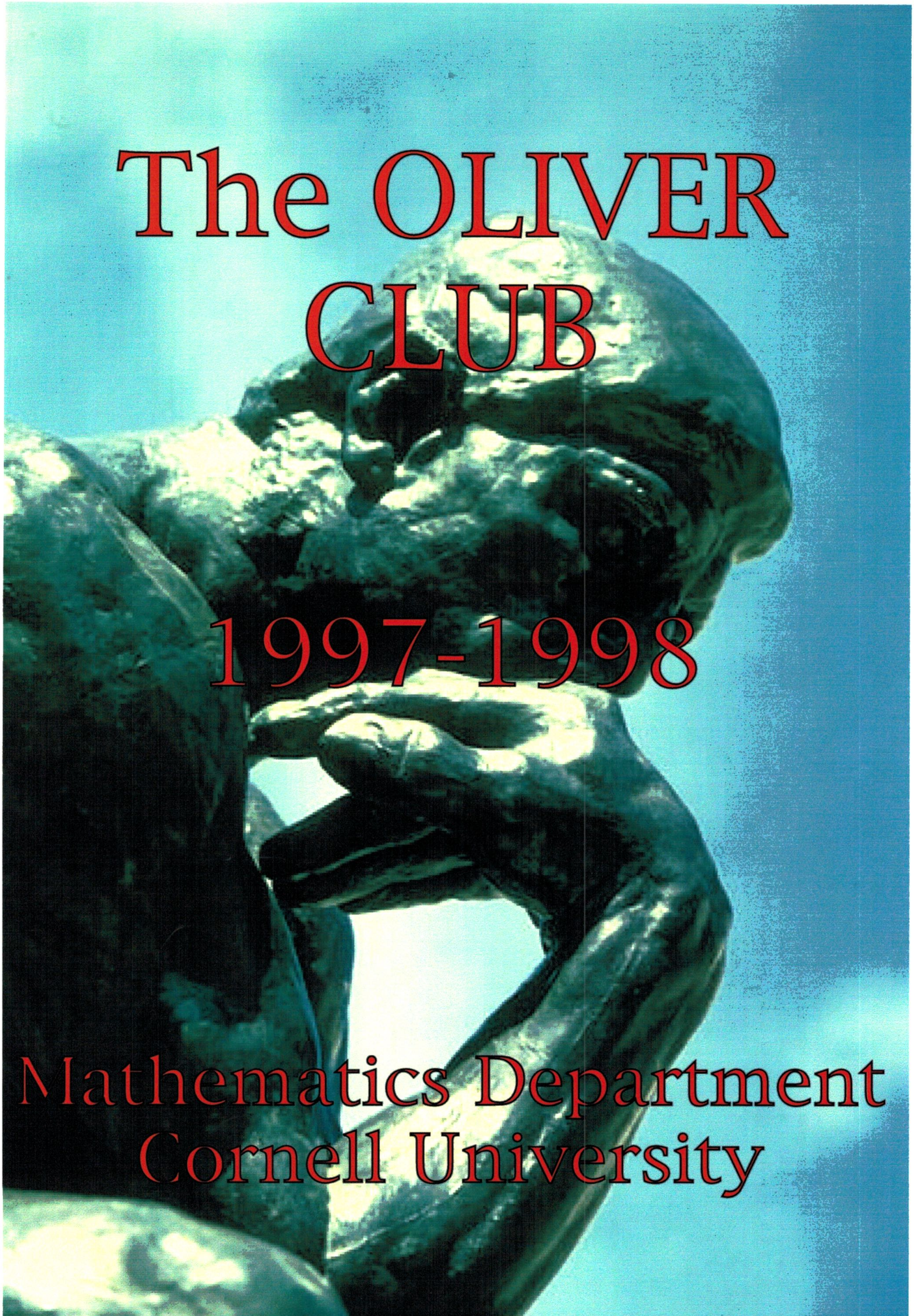
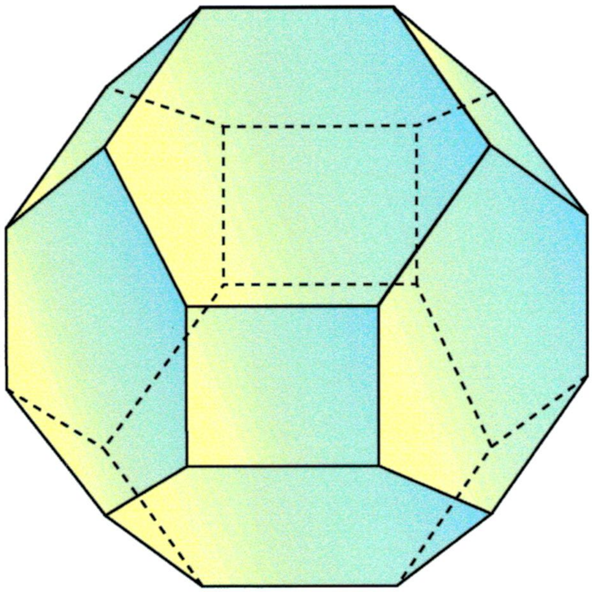
The OLIVER CLUB

1997-1998

Mathematics Department
Cornell University





The OLIVER CLUB

presents

Louis Billera

Cornell University

Thursday, September 11
4:15 p.m.
328 White Hall

Counting faces in polytopes, spheres and hyperplane arrangements

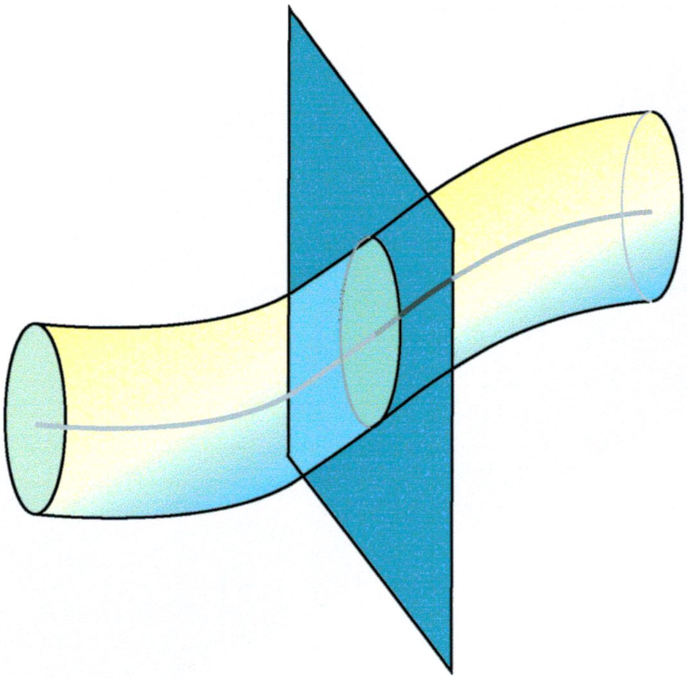
The problem of determining the relations between the numbers of faces of all dimensions in convex polytopes is one that has amused mathematicians for hundreds of years. In recent years, much progress has been made on this problem for the interesting special class of simplicial polytopes. This involved an infusion of methods from diverse areas of mathematics .

Still the general situation remains quite unsettled. The question of determining the possible face counts for general convex polytopes has been completely answered only in dimension 3 (done 90 years ago!). There is not even a conjecture as to the form of a complete solution in dimension 4.

Currently, a variety of new approaches are being developed, both for the general case and for interesting special cases, such as cubical polytopes or spherical subdivisions induced by arrangements of linear hyperplanes.

The lecture will give an overview of these developments suitable for a general audience.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Peter Heinzner

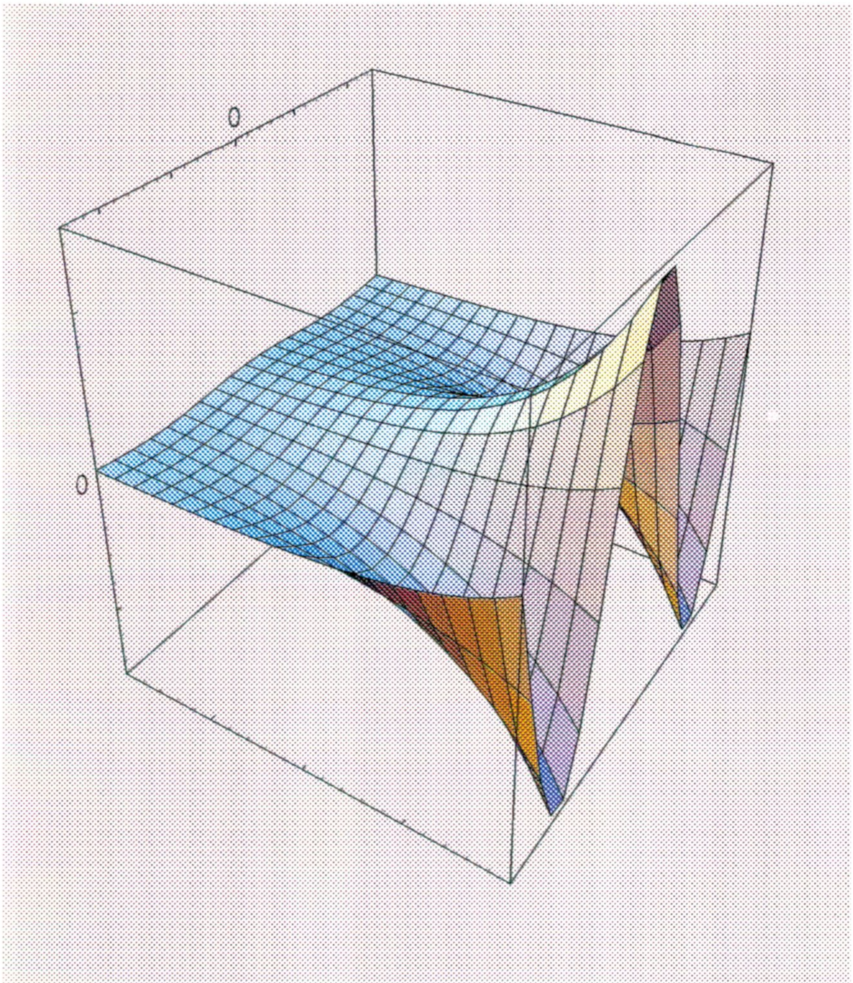
Brandeis University

Thursday, September 18
4:15 p.m.
328 White Hall

Complex geometry of proper actions

Roughly speaking a smooth action of a Lie group G on a manifold is said to be proper if G is a group of isometries with respect to some Riemannian metric. We will begin with some basic properties of proper actions. The situation where G preserves a Kähler metric on a complex manifold will be discussed more closely. Such an action is quite often generated by Hamiltonian vector fields. The relevance of this fact for the complex geometry of the underlying space will be the topic of the last part of the talk.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Laurent Saloff-Coste

C.N.R.S.
Université Paul Sabatier
Toulouse, France

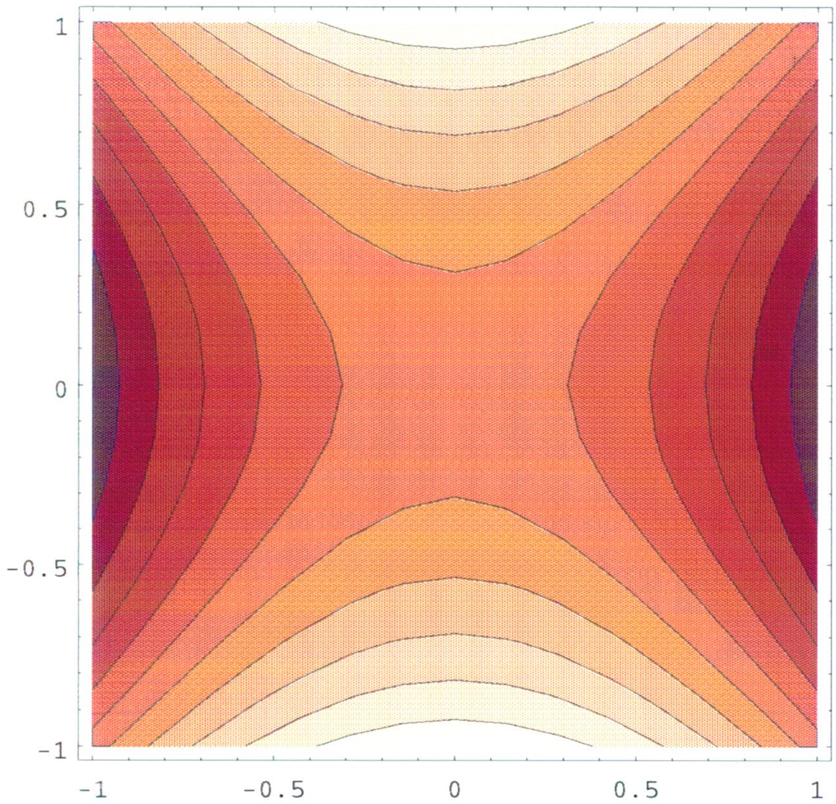
Thursday, September 25
4:15 p.m.
328 White Hall

Harnack Inequalities and Geometric Applications

The Harnack inequality, in its most classical form, asserts that there exists a constant C such that for any ball B(0,R) in Euclidean space, any positive harmonic function u in B(0,R) and any points x,y in B(0,R/2), the ratio u(x)/u(y) is bounded by C. This elliptic inequality has been generalized over the years to various partial differential equations including certain parabolic equations.

This talk will focus on geometric aspects of Harnack inequalities and their applications to Liouville type theorems and heat kernel estimates.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Peter Li

University of California
Irvine

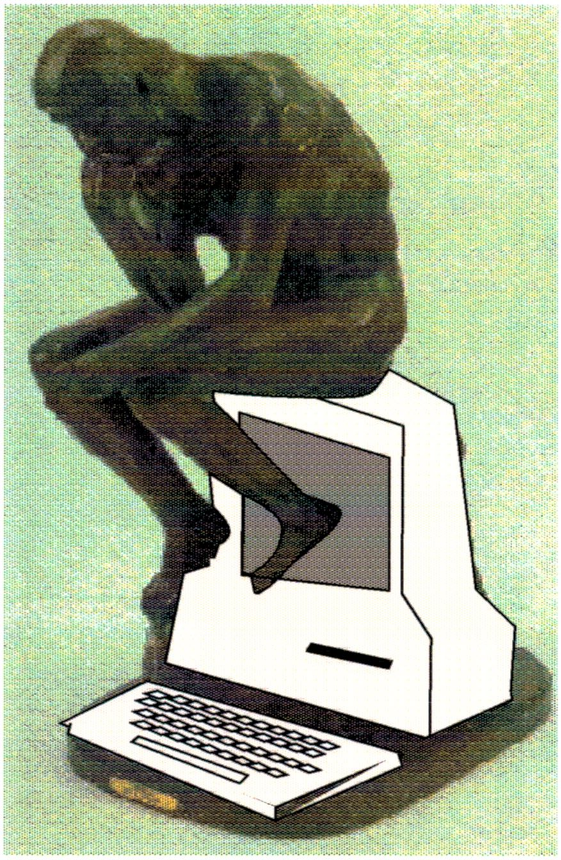
Thursday, October 2
4:15 p.m.
328 White Hall

Polynomial growth harmonic functions and harmonic maps

Abstract

Harmonic functions are important analytical objects that relate analysis to geometry. In this talk, I will give a brief overview of the theory of harmonic functions that have polynomial growth rate. In particular, I will describe a short, simple proof of the result that the space of degree d polynomial growth harmonic functions is finite dimensional. I will then indicate how the same type of argument can be applied to study the structure of the set of harmonic maps (which are the nonlinear analog of harmonic functions) into a simply connected manifold with non-positive curvature. The talk is aimed at general audience.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Sergei Artemov

Cornell and Moscow

Thursday, October 9
4:15 p.m.
328 White Hall

Proof Polynomials

Abstract

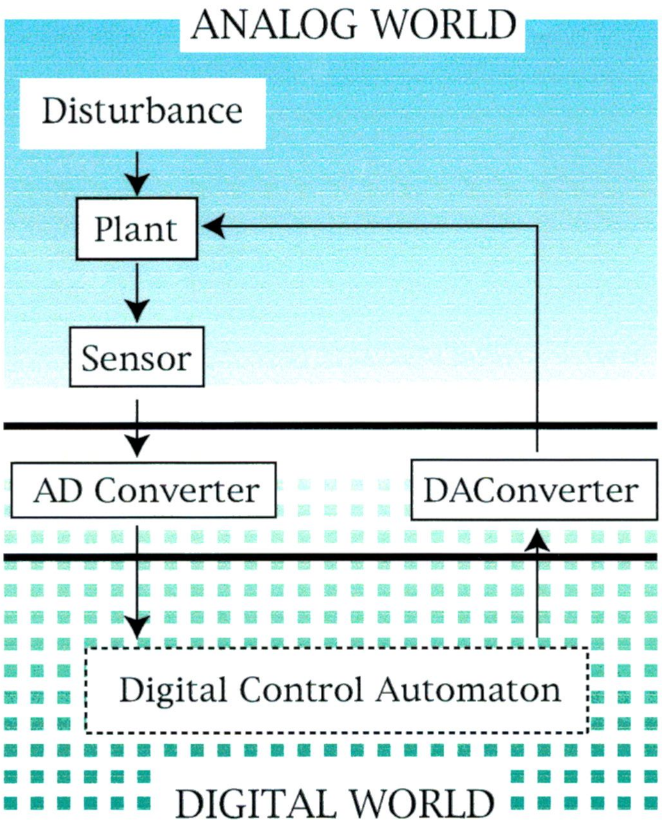
Provability is one of the central concepts of mathematical logic and related areas.

In 1933 Gödel introduced an axiomatic system D for an absolute notion of provability (i.e. not depending on the formalism chosen), but left open the problem of finding a mathematical provability model for it. The famous formal provability predicate which first appeared in the Gödel Incompleteness Theorem does not do this job: the logic of formal provability is not compatible with D.

As was discovered in 1995, this defect of the formal provability predicate can be bypassed by adding an extra dimension and replacing hidden quantifiers over proofs by *proof polynomials* in a certain finite basis.

The resulting *Dynamic Logic of Proofs* provides an exact mathematical model for major constructions in pure and applied logic based on the concept of provability, including modal logic, intuitionistic logic with its Brouwer-Heyting-Kolmogorov interpretation, lambda calculus and modal lambda calculus. In particular, the Gödel provability logic D becomes a forgetful projection of the Dynamic Logic of Proofs and thus meets a desired provability model. It solves the Gödel problem of 1933.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Anil Nerode

Cornell University

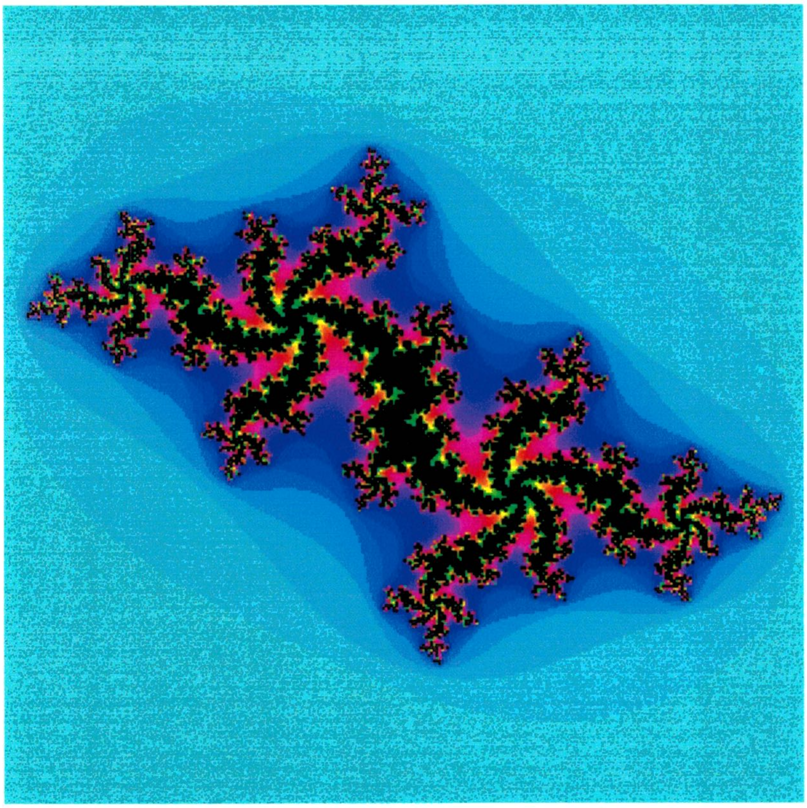
Thursday, October 16
4:15 p.m.
328 White Hall

Hybrid Systems and Distributed Autonomous Control

In this talk I will describe joint work with Dr. Wolf Kohn, in which we describe strategies for controlling systems of interacting industrial processes using digital computers. There are applications to air transport control, video compression and distributed routing, among others. This work is being commercialized by Hybrithms Corporation.

We model each system as a differentiable manifold. Allowable trajectories are those whose tangents are in certain truncated cones determined by the data of the problem. A trajectory represents ideal performance when it minimizes a Lagrangian integral, representing cost, over all allowable trajectories. We use a large dose of automata theory, relaxed variational calculus and classical differential geometry to produce finite state automata which guarantee near optimal control.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Mikhail Lyubich

S.U.N.Y. at Stony Brook

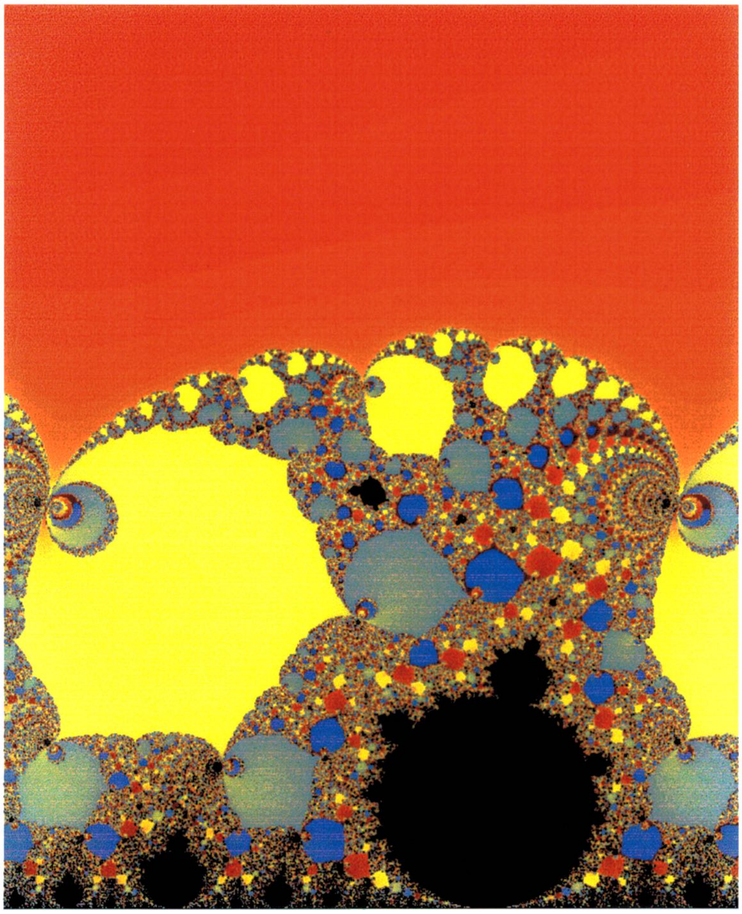
Thursday, October 23
4:15 p.m.
328 White Hall

Proof of the Renormalization Conjecture

Abstract

The "Feigenbaum Universality Law" discovered about 20 years ago has fascinated both physisists and mathematicians. Physically, it gives a prediction of the transition parameter from "laminar" to "turbulent" regimes. Mathematically, it happened to be a deep problem on the borderline of dynamics, analysis and geometry. In the talk, we plan to discuss mathematical structures behind this problem which have recently led to its solution.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

John Hubbard

Cornell University

Thursday, October 30
4:15 p.m.
328 White Hall

Geometric Limits of Polynomials

Abstract

What is the limit of the subgroup $\epsilon\mathbf{Z}$ of $\mathbf{R}$ as $\epsilon \to 0$?

Answer: $\mathbf{R}$ (not 0 !).

More generally, if $f_\epsilon(z) = z^2 + z + \epsilon$, what is the limit of the dynamical system $(f_\epsilon^{\,n})$, $n = 1,2,\ldots$?

The answer leads to "enriched polynomials," which can be used to show that the boundary of the Mandelbrot set has Hausdorff dimension equal to 2.

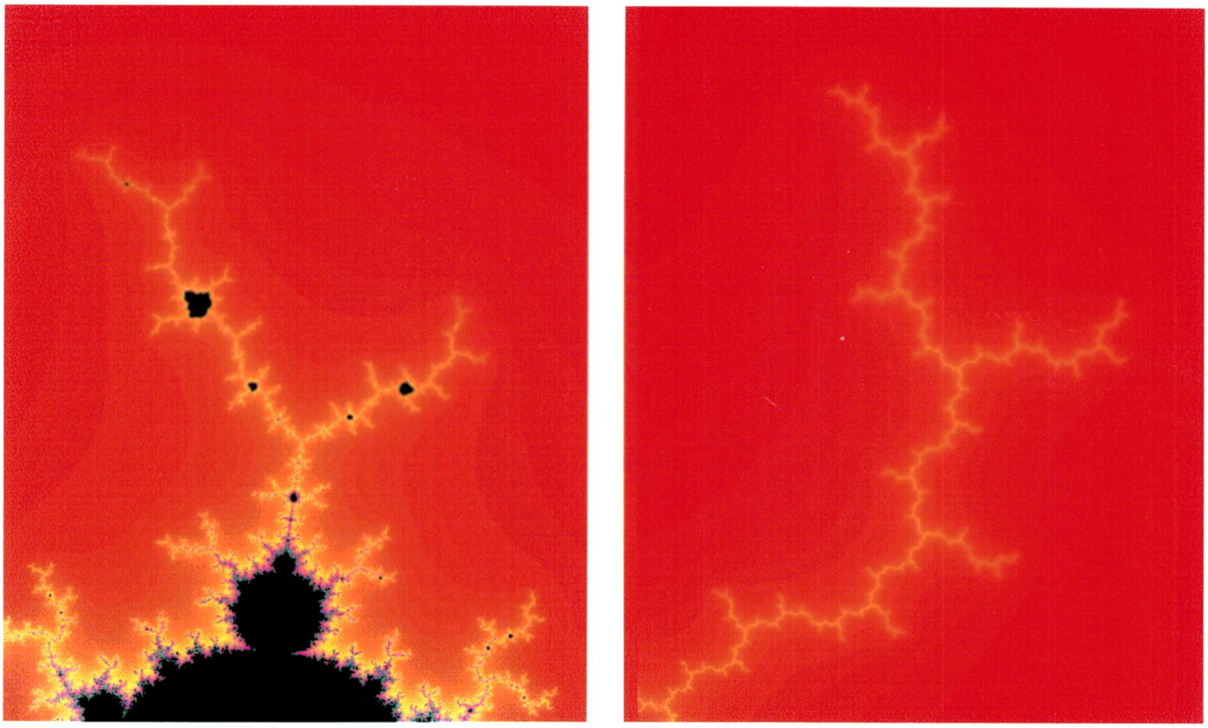
Refreshments will be served at 3:45
in the math department lounge.

The OLIVER CLUB
presents

Xavier Buff

Cornell University
328 White Hall

Thursday, November 6
4:15 p.m.

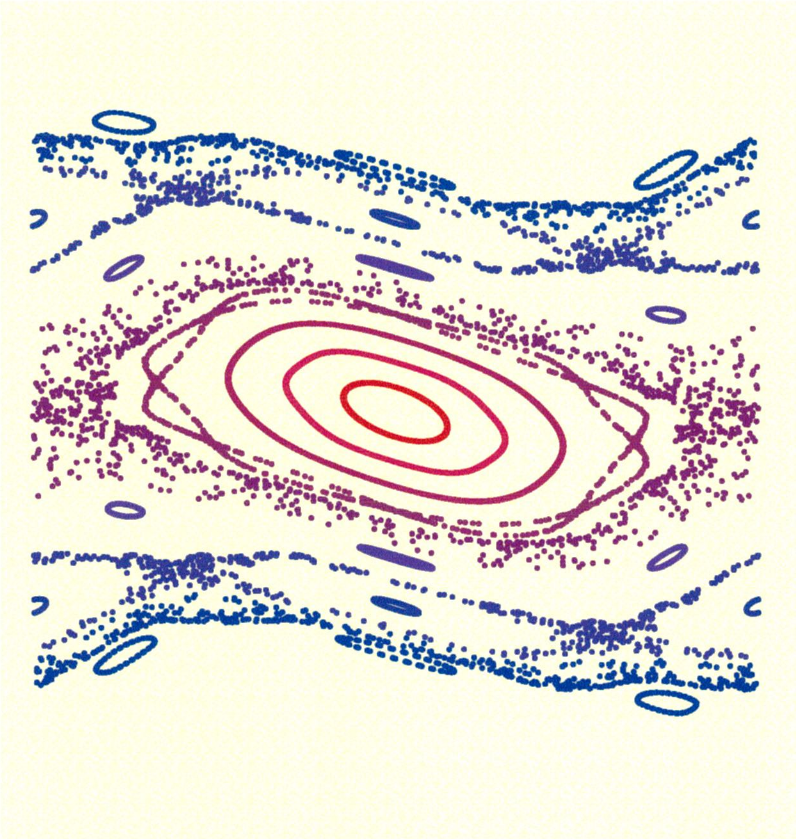


Holomorphic motions in dynamical systems

Abstract

Most of the results on the Mandelbrot set have been obtained by using its similarities with Julia sets: "plow in the dynamical plane and harvest in the parameter plane." The most powerful tool for transferring results from Julia sets to the Mandelbrot set is the use of holomorphic motions.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

John Franks

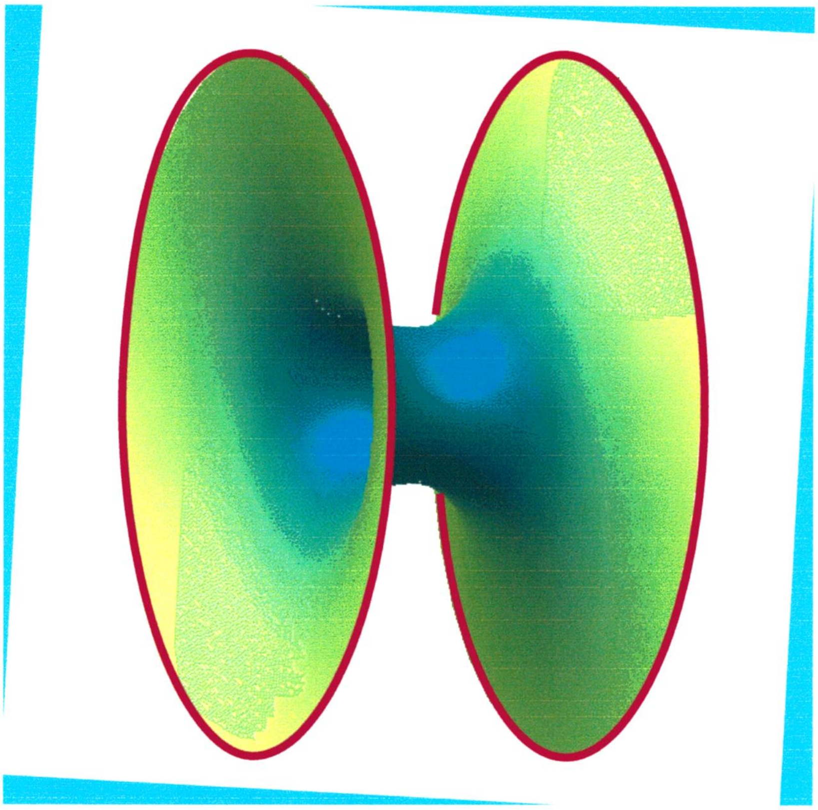
Northwestern University

Thursday, November 13
4:15 p.m.
328 White Hall

Area preserving surface homeomorphisms and rotation vectors

In this talk I will consider the concepts of rotation number and rotation vector for area preserving homeomorphisms of surfaces and their applications. For an annulus **A** the rotation number of a point x in **A** represents an average rate at which the iterates of x rotate around the annulus. More generally the rotation vector takes values in the one dimensional homology of the surface and represents the average "homological displacement" of an orbit. Applications to the analysis of dynamical properties of surface homeomorphisms include a proof of the "Arnold conjecture" on the existence of fixed points.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

José F. Escobar

Cornell University

Thursday, November 20
4:15 p.m.
328 White Hall

Some Isoperimetric Problems

The classical isoperimetric inequality relates the area A of a domain in the plane with the perimeter L of the boundary:

$$L^2 \geqslant 4 \pi A$$

and equality holds only if the domain is a disc. Isoperimetric problems are simply variational problems with constraints, whose name derives from the fact that the above inequality corresponds to the first example of such a problem: maximize the area of a domain under the constraint that the perimeter of the boundary is fixed.

The problems of finding eigenvalues for the Laplacian and finding Sobolev inequalities can also be characterized as isoperimetric problems. In this lecture I will discuss the relation among these isoperimetric problems and indicate their relevance in differential geometry and in partial diffferential equations.

Refreshments will be served at 3:45
in the math department lounge.

The OLIVER CLUB

presents

Curt McMullen

U.C. Berkeley
and
Harvard University

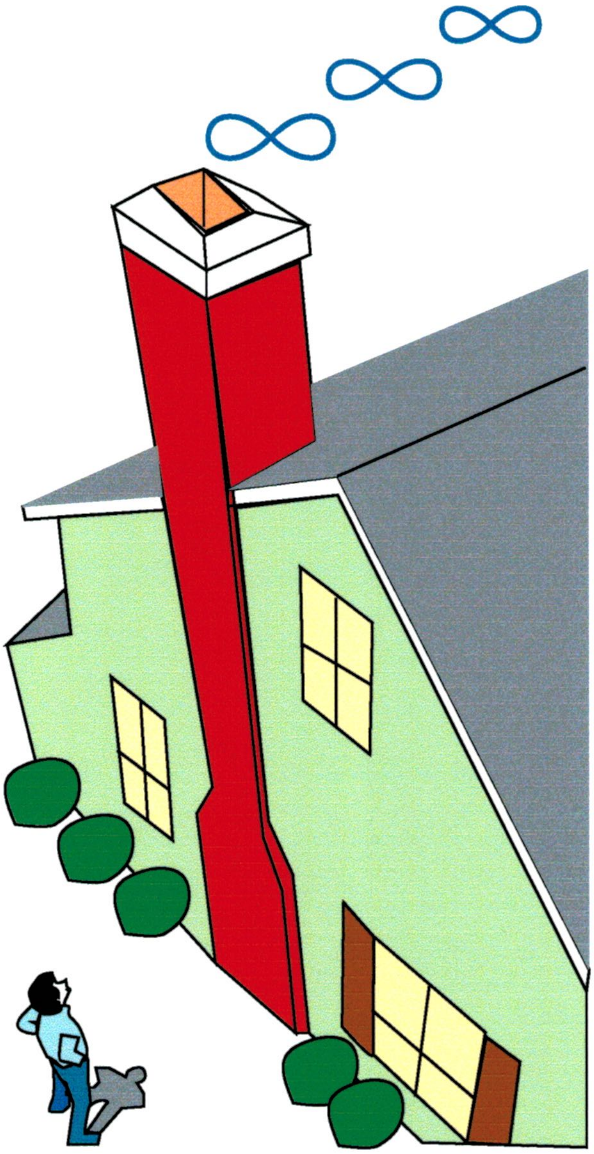
Thursday, January 22
4:15 p.m.
328 White Hall

Exotic hyperbolic 3-manifolds

A hyperbolic 3-manifold M(X,Y) can be associated in a natural way to a pair of complex tori. What happens when the tori degenerate? In the 1970s Bers showed an exotic action of Z*Z on the Riemann sphere arises in the limit. The chaotic locus for this action is a fractal tree of Hausdorff dimension two but measure zero. Thurston and Minsky showed the geometry of the corresponding 3-manifold can be simply described by an irrational number.

This talk will present work leading to some of the first explicit constructions of these exotic 3-manifolds. The construction works for irrational numbers such as the golden mean, and was suggested by an analogy with renormalization of dynamical systems.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Nikola Lakic

Cornell University

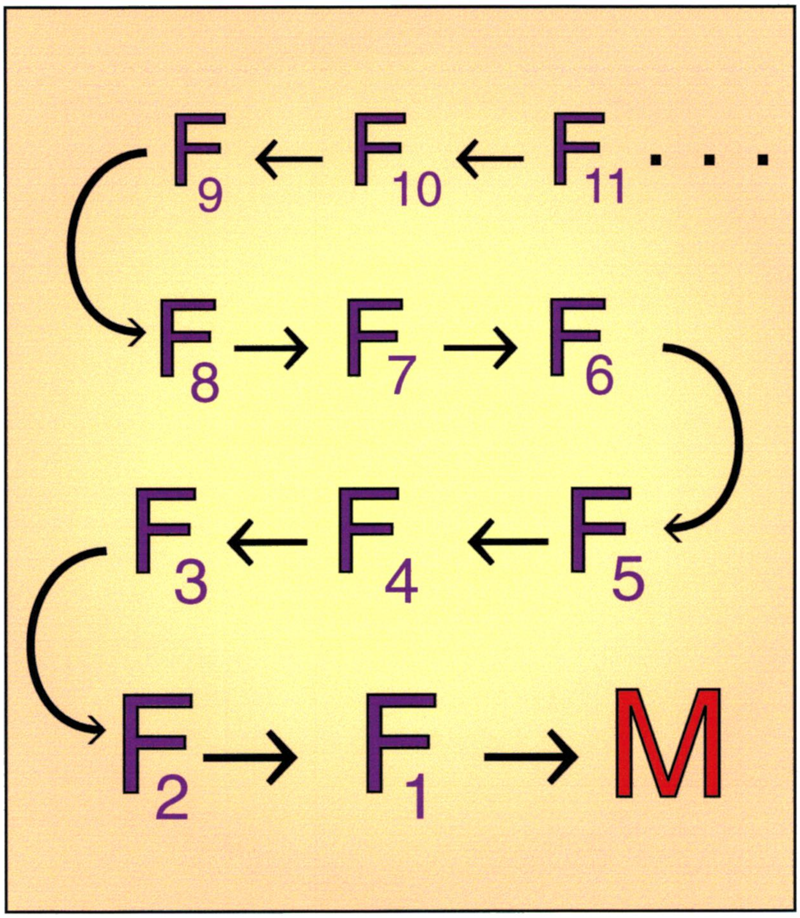
Thursday, January 29
4:15 p.m.

328 White Hall

Substantial boundary points of plane domains

Let f be a homeomorphism of the unit circle onto itself. We will study the conformal distortion of extensions of f to the open unit disk (global distortion), to neighborhoods of the unit circle (boundary distortion), and to neighborhoods of points on the unit circle (local distortion). We will discuss a domain in the plane called *Strebel's chimney*. It provides an f with exactly one point where all three distortions are the same. It is a well known theorem that there is always at least one "substantial point" where the local distortion equals the boundary distortion. We extend this theorem to all plane domains.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Irena Peeva

Massachusetts Institute of Technology

Thursday, February 5
4:15 p.m.
328 White Hall

Free resolutions

Hilbert introduced the idea to describe the structure of a module M by an exact sequence of free modules and maps between them. Such a sequence is called a free resolution of M. In essence, constructing a resolution consists of repeatedly solving systems of linear equations. Recent computational methods have made it possible to compute free resolutions by computer.

In some important cases there exists a minimal free resolution of M. It is unique up to an isomorphism and is contained in any free resolution of M. The invariants of M are closely related to the properties of its minimal free resolution.

The talk will be a survey on open questions about structural and numerical properties of minimal free resolutions.

Refreshments will be served at 3:45
in the math department lounge.

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The OLIVER CLUB

presents

Adrian Lewis

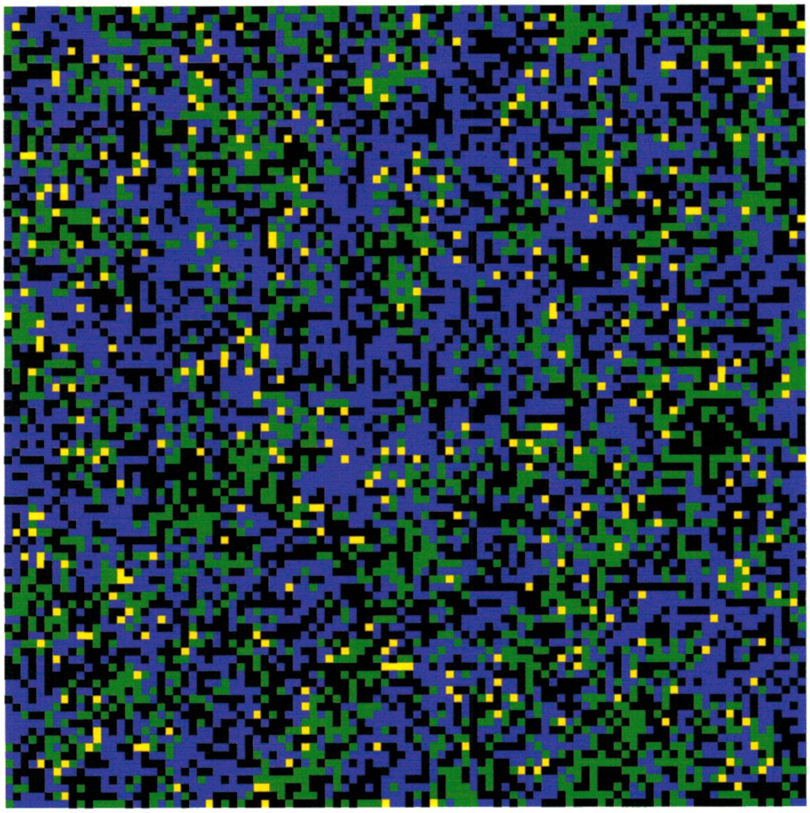
University of Waterloo

Thursday, February 12
4:15 p.m.
328 White Hall

Eigenvalues, hyperbolic polynomials, and the Kostant Convexity theorem

Any permutation-invariant convex function of the eigenvalues of a symmetric matrix is convex (Davis, 1957). Analogously, von Neumann's famous characterization of unitarily invariant matrix norms (1933) studies permutation-invariant gauge functions of the singular values. I discuss two elegant unifications of these fundamental results. The first considers, for example, the determinant of a symmetric matrix as a homogeneous polynomial which is "hyperbolic" with respect to the identity matrix (the characteristic polynomial always having only real roots). The second, more sophisticated approach uses the Kostant Convexity Theorem (1973) from semi-simple Lie theory. I sketch both approaches using the symmetric matrix case as a guiding example, and outline how the second framework gives a powerful duality theory subsuming von Neumann's original argument.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Rick Durrett

Cornell University

Thursday, Feb. 19
4:15 p.m.
328 White Hall

When is space important in ecological modeling?

Durrett and Levin (1994) proposed that the behavior of stochastic spatial models could be inferred from the mean field ordinary differential equation which results from assuming the system is "homogeneously mixing." In brief, space is not important if the ODE has a single attracting fixed point but is important if there is more than one stable fixed point or periodic orbits. We will present theorems, conjectures, and videotaped simulations in support of this picture.

**Refreshments will be served at 3:45
in the Math Department Lounge**

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3 + 1 = ?

The OLIVER CLUB

presents

Richard A. Shore

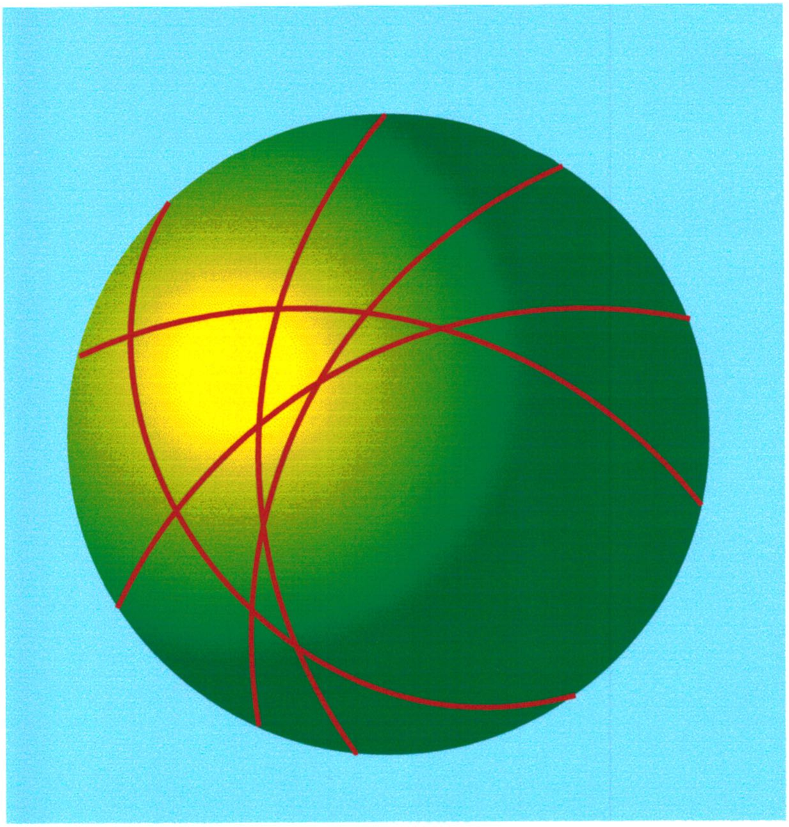
Cornell University

Thursday, February 26
4:15 p.m.
328 White Hall

Computable Structures: Presentations Matter

By a *structure* we mean a set together with a specified set of relations and functions; for example, a group, ring or field is a structure, as is a linear ordering or Boolean algebra. A structure is *computable* if its underlying set together with all its specified relations and functions are computable. We are interested in whether additional relations or functions on the structure are computable; for example, can one compute whether elements of a field are algebraically independent over a subfield, or compute the center of a group, or the successor in a linear ordering? In general, the answer to these questions depends on the presentation of the structure we are given. We will consider conditions that guarantee that additional relations computable in one (computable) presentation of the structure are computable in any computable presentation. On the other hand, we will describe situations in which the complexity of such relations and functions can vary widely as the computable presentation of the given structure changes.

Refreshments will be served at 3:45
in the math department lounge.



The OLIVER CLUB

presents

Ken Brown

Cornell University

Thursday, March 5
4:15 p.m.
428 White Hall

Geometry and probability in three dimensions

A collection of n great circles on the unit sphere in $\mathbf{R}^3$ decomposes the sphere into vertices, edges, and regions (2-cells). If the great circles are in general position, there are $n(n-1)$ vertices, $2n(n-1)$ edges, and $n(n-1)+2$ regions. There is a natural random walk on the regions: If we are in a region C, we pick a vertex at random and move to the region adjacent to that vertex which is closest to C.

In this talk I will describe joint work with Louis Billera and Persi Diaconis in which we combine tools from geometry, topology, and probability to give a complete analysis of the random walk (stationary distribution, rate of convergence, eigenvalues). The formula for the stationary distribution seems quite surprising and mysterious at present. At the end of the talk I will put the results into a broader context (random walks associated to hyperplane arrangements and buildings) and mention a number of situations where such walks arise naturally.

Refreshments will be served at 3:45
in the Math Department Lounge

The OLIVER CLUB

presents

Barbara Shipman

University of Rochester

Thursday, March 12
4:15 p.m.
328 White Hall

The geometry of momentum mappings on generalized flag manifolds: connections with a dynamical system, quantum mechanics, and the dance of the honeybee

The momentum mapping on a generalized flag manifold sends each orbit of the complex diagonal torus to a convex polytope whose real dimension is the complex dimension of the orbit. This mapping is useful in understanding the types of singularities that arise in a completely integrable system known as the Toda lattice, and it also has connections with quantum mechanics and biology. In the flag manifold of Sl(3,C), the images under the momentum mapping of certain orbits of a 1-dimensional affine group correspond to patterns that arise in the dance that honeybees use to communicate the location of a food source. The geometry contains parameters for both the distance and direction to the source and reflects the discontinuous change in form from the "waggle dance," used for distant sources, into the "round dance," for nearby sources. While much is known about how distance and direction are encoded in the dance, little is understood about how a bee processes information biologically to produce the dance. It is known that the dance incorporates physical influences including polarized light, gravity, and the earth's magnetic field. The mathematics has a quantum mechanical interpretation that suggests that the bees may also be perceiving certain quantum fields and using them in the choreography of the dance.

Refreshments will be served at 3:45
in the Math Department Lounge

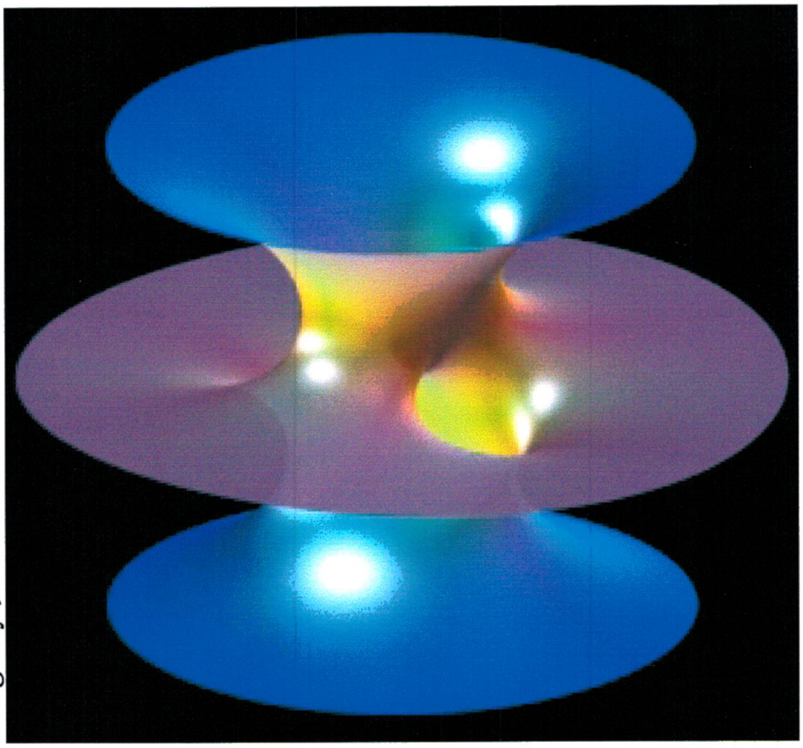


Image by James T. Hoffman

The OLIVER CLUB

presents

Richard Schoen

Stanford University

Thursday, March 26
4:15 p.m.
328 White Hall

A Plateau problem in complex geometry

This will be a lecture, intended for a general audience, about the variational problem of finding least volume lagrangian submanifolds of Kahler or symplectic manifolds. Motivations for this problem arise in Kahler geometry, mirror symmetry, and nonlinear elasticity. We will describe the connections with special lagrangian geometry. We will outline the progress which has been made on this variational problem, and describe what we see as the main obstacles toward a better understanding.

Refreshments will be served at 3:45
in the Math Department Lounge

The OLIVER CLUB

presents

Paul Sally

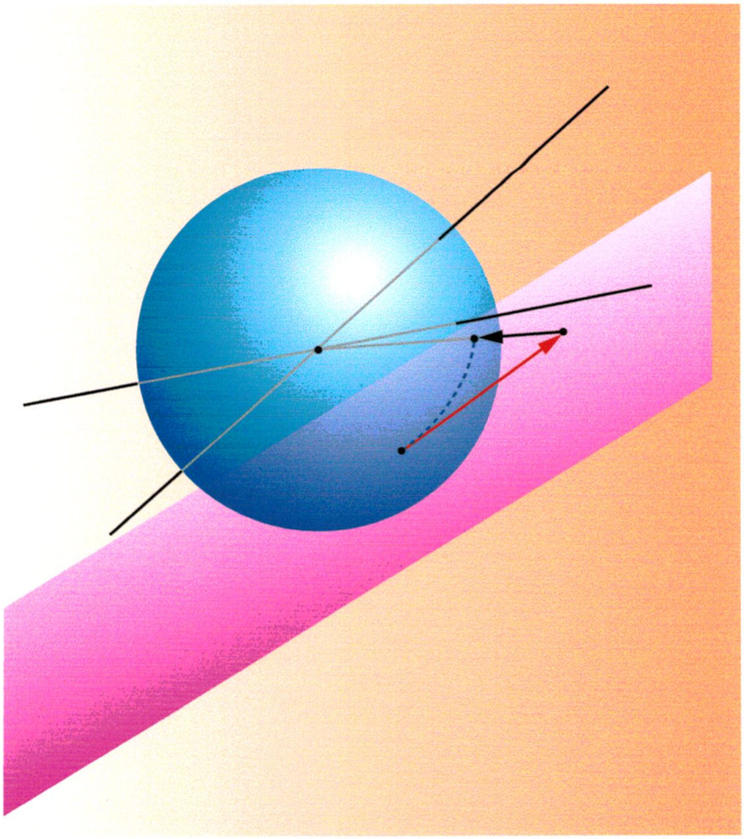
University of Chicago

Thursday, April 2
4:15 p.m.
328 White Hall

Harmonic analysis on p-adic general linear groups

The p-adic numbers were introduced by number theorists to solve systems of diophantine equations. Analysis on the p-adic numbers comes into work of Tate and others in problems of class field theory. In recent years, non-abelian class-field theory requires analysis on p-adic matrix groups (Langlands Program). This talk introduces the ideas above for a general audience and leads to a glimpse of Haris-Chandra's great work (and beyond).

Refreshments will be served at 3:45
in the Math Department Lounge



The OLIVER CLUB

presents

Jim Renegar

Cornell University

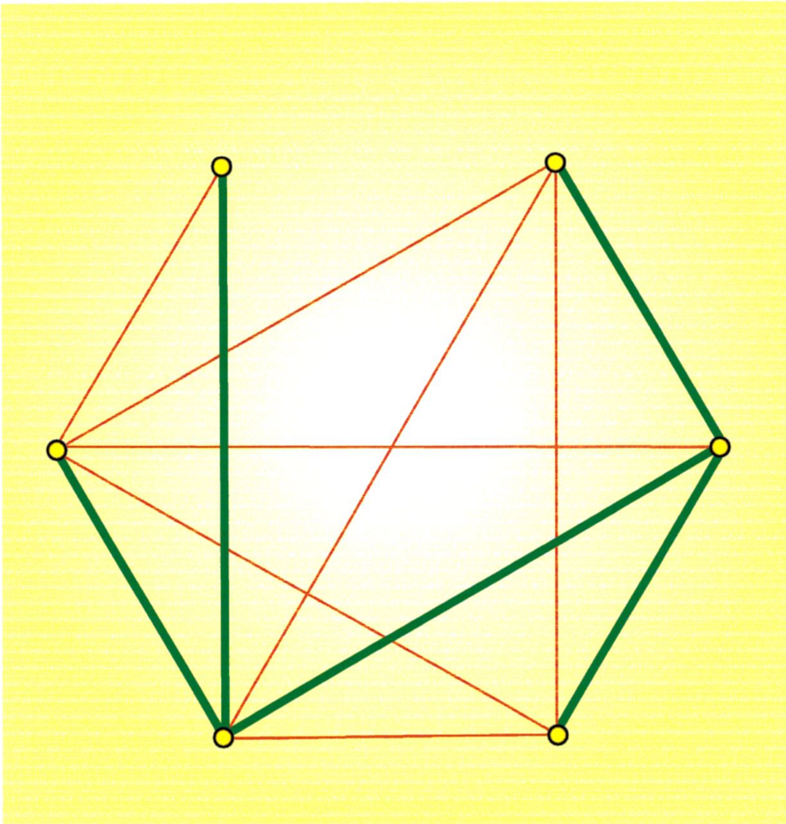
Thursday, April 9
4:15 p.m.
328 White Hall

How to explain the efficiency of root-finding algorithms?

Algorithms for approximating roots of polynomials have long formed a cornerstone of numerical analysis. In recent years, emphasis has shifted to systems of polynomials, and homotopy methods have taken center stage. Unlike algebraic algorithms, such as some relying on the computation of resultants, these numerical algorithms, based on Newton's method, can take arbitrarily long to approximate roots, i.e., have unbounded worst-case complexity. However, in practice, the numerical algorithms have proven to be far more efficient than the algebraic ones, especially for systems having more than a few variables.

Much effort has gone into attempts to develop a theory explaining the efficiency. I will present some of the main concepts and results in the theory, culminating with a discussion of the proof techniques recently used by Shub and Smale to show that, on average, a root of a system of polynomials can be computed in polynomial-time. I will indicate some significant problems that remain unsolved.

Refreshments will be served at 3:45
in the Math Department Lounge



The OLIVER CLUB

presents

Richard Stanley

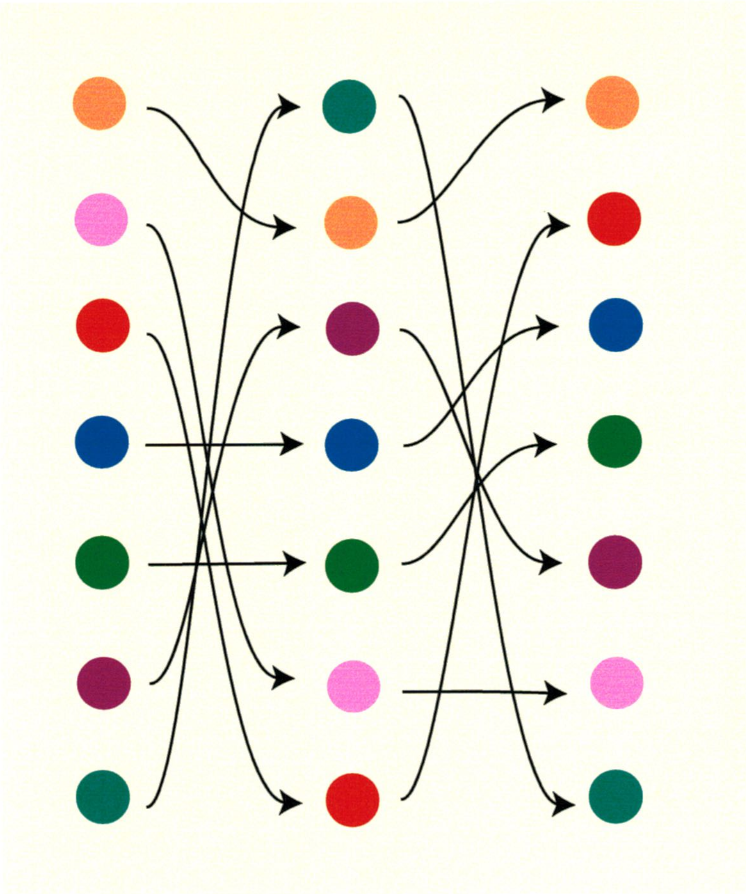
Massachusetts Institute
of Technology

Thursday, April 16
4:15 p.m.
328 White Hall

Spanning trees and a conjecture of Kontsevich

In connection with the evaluation of some integrals related to formal quantum field theory, Kontsevich conjectured that the number $c_G(q)$ of zeros over the finite field $\boldsymbol{F}_q$ of a certain multivariate polynomial associated with the spanning trees of a graph G was a universal polynomial in q (depending only on G, and not on the characteristic of the field). We will discuss the connection between this conjecture, the Matrix-Tree Theorem, and the enumeration of certain nonsingular symmetric matrices over $\boldsymbol{F}_q$. For certain classes of graphs, the function $c_G(q)$ can be explicitly evaluated. Although Kontsevich's conjecture itself remains open at the moment, we will explain why we believe that it is false.

Refreshments will be served at 3:45
in the Math Department Lounge



The OLIVER CLUB
presents

Phil Hanlon

University of Michigan

Thursday, April 23
4:30 p.m.
328 White Hall

Combinatorics and Lie Theory

The free Lie algebra LIE(n) is a beautiful, simple object with connections to Lie group cohomology and representation theory of the symmetric group. This will be an expository account of this set of ideas.

Refreshments will be served at 3:45
in the Math Department Lounge

The OLIVER CLUB
and
the CORNELL TOPOLOGY
FESTIVAL
present

Daryl Cooper

University of California
Santa Barbara

Friday, May 1
4:30 p.m.
Kaufman Auditorium

Surfaces Subgroups Sometimes Survive Surgery

Suppose M is a 3-manifold and N is M with an open solid torus neigborhood of a simple closed curve deleted. Then M is said to be obtained by *Dehn-filling* N. It has long been known that every closed orientable 3-manifold M can be obtained from the complement of finitely many disjoint solid tori in the 3-sphere by suitable Dehn-fillings. Thurston has shown that most 3-manifolds are hyperbolic. Earlier, Haken and Walhausen developed an extensive theory of 3-manifolds which contain an essential embedded surface.

We show that all but finitely many Dehn fillings of a one-cusped hyperbolic 3-manifold contain a surface group. The method involves quasi-Fuchsian subgroups and analysing certain convex sets.

**Refreshments will be served at 3:45
in the Math Department Lounge**