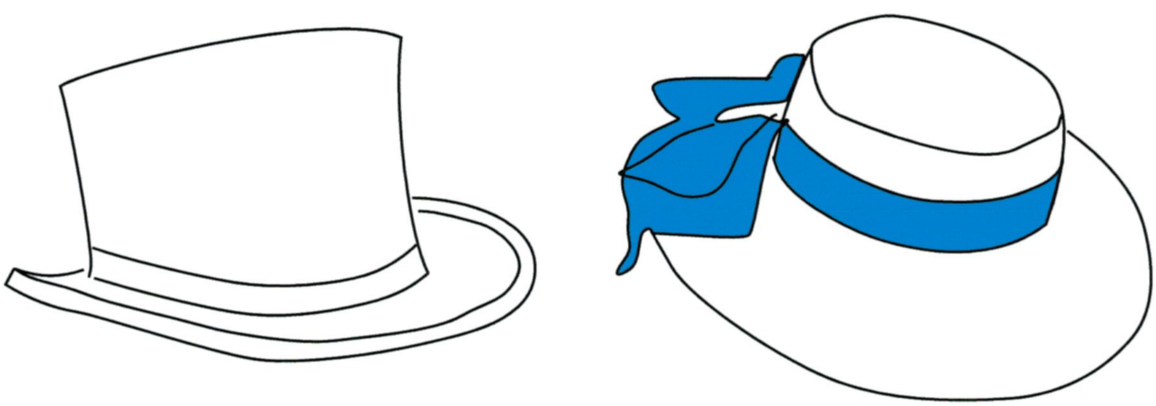
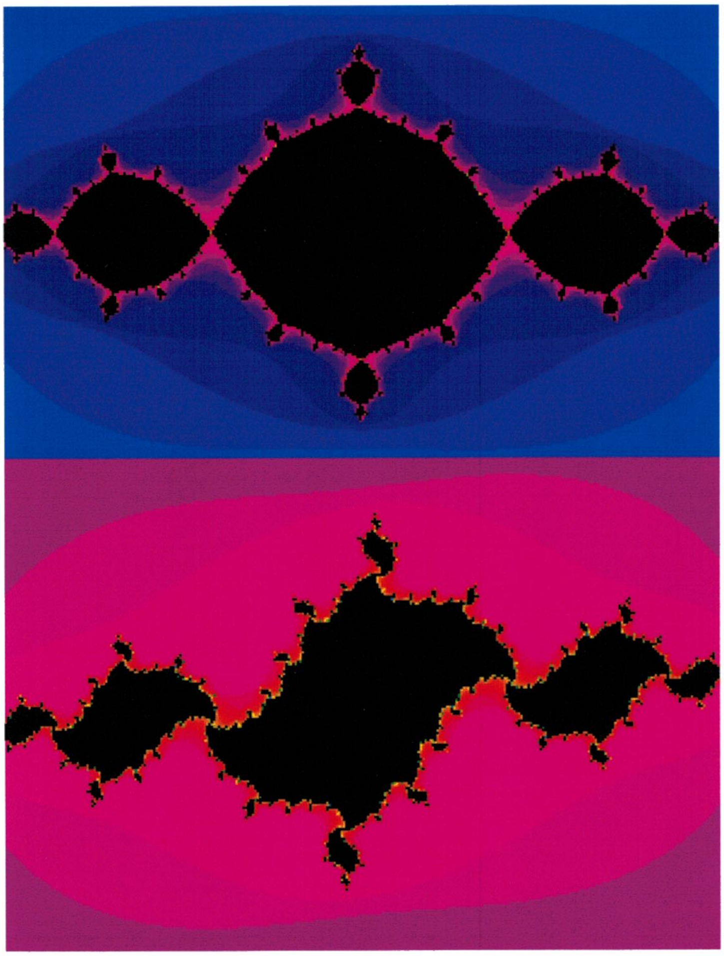
# The OLIVER CLUB

# 1998-1999



Mathematics Department
Cornell University



The OLIVER CLUB

presents

**Greg Buzzard**

of
Cornell University

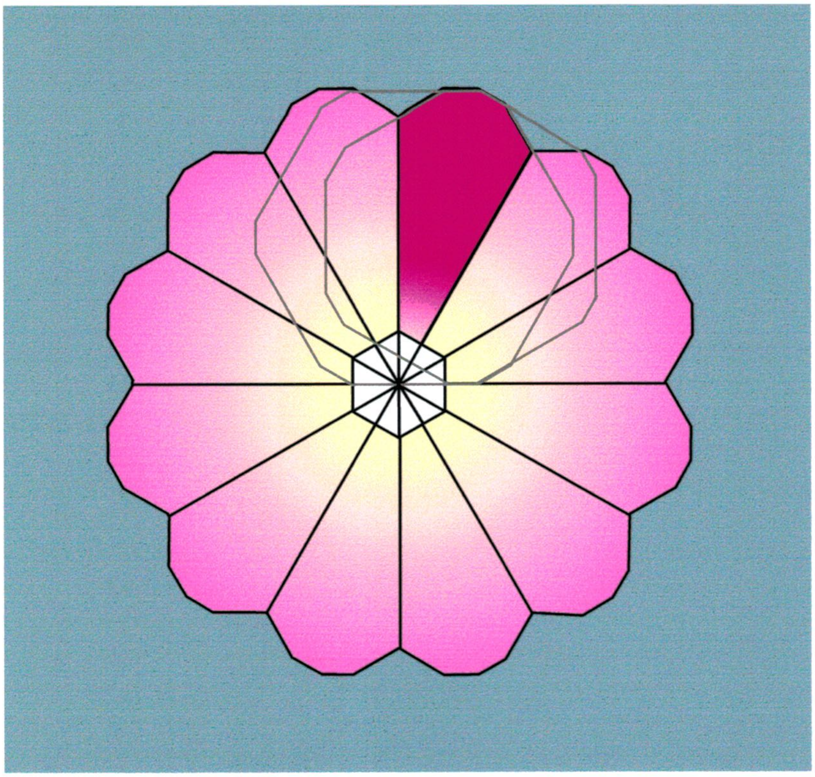
Thursday, September 10
4:15 p.m.
428 White Hall
Cornell University

# Dynamical stability for polynomial diffeomorphisms of $\mathbb{C}^2$

In the study of dynamics, a natural subject to consider is the stability of the system being studied; i.e., does a small change to the system lead to a correspondingly small change in the behavior of that system? The precise formulation of this idea of stability is given in topological terms. On the other hand, for the case of $C^1$ diffeomorphisms of a compact manifold, this topological notion is known to be equivalent to a few conditions which are analytical in nature.

In this talk I will present these ideas more fully and discuss the ways in which they relate to polynomial diffeomorphisms of complex 2-space.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB

presents

**Reyer Sjamaar**

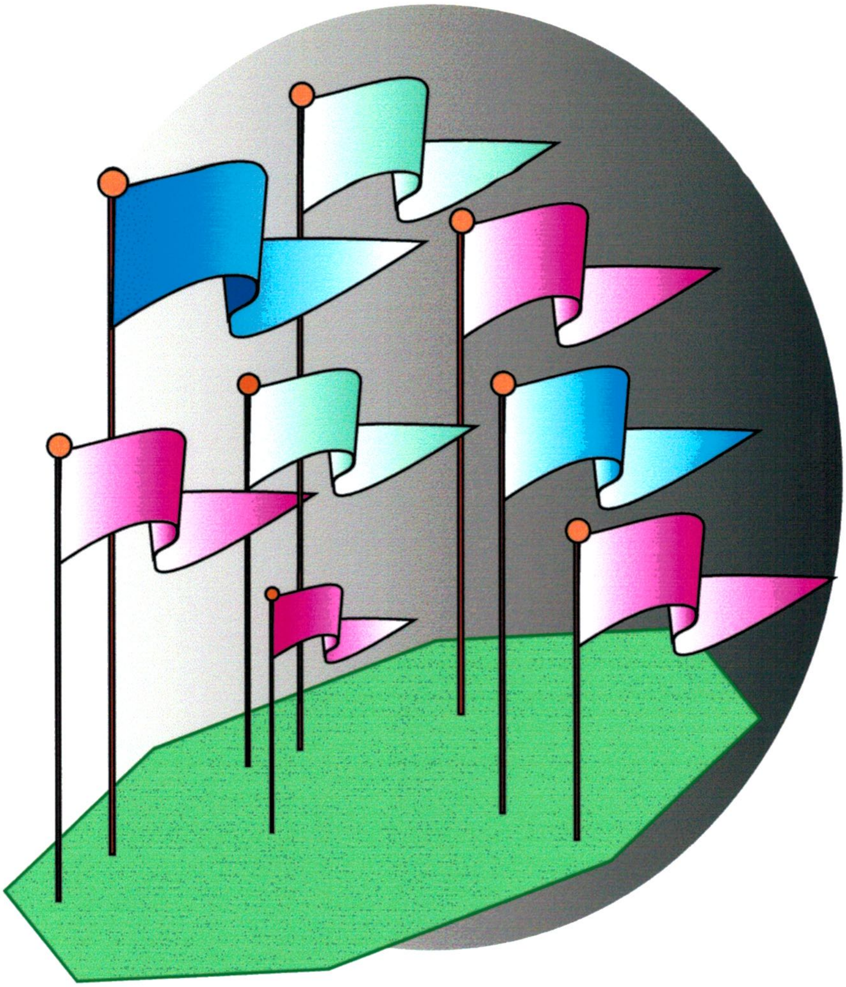
Cornell University

Thursday, Sept. 17
4:15 p.m.
328 White Hall

# Linear inequalities and Schubert cycles

Given the eigenvalues of two Hermitian matrices, what are the possible eigenvalues of their sum? Partial answers to this question were found by H. Weyl and many others in the form of inequalities relating the eigenvalues of A, B and A+B. A complete answer was given recently by A. Klyachko, who gave an exhaustive list of all possible inequalities. I will discuss Klyachko's work and a generalization due to A. Berenstein and myself.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB

presents

**Lou Billera**

of

Cornell University

Thursday, September 24

4:15 p.m.

328 White Hall

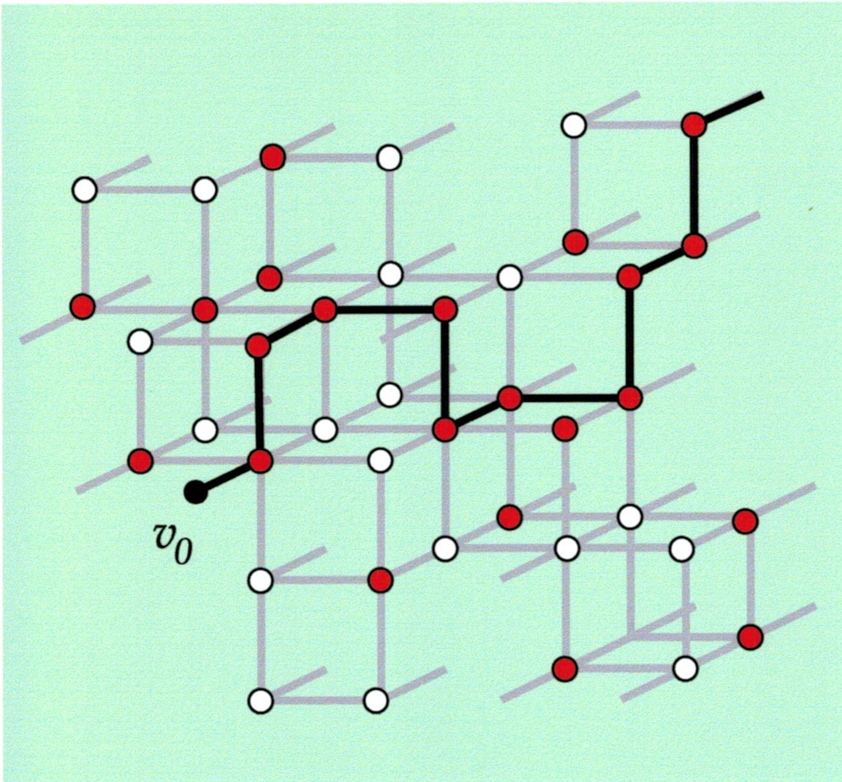
Cornell University

# Enumerating flags in polytopes

For a convex polytope P, the number $f_S(P)$ counts the number of flags $F_1 \subset \ldots \subset F_k$ of faces of P, where $S = \{ \dim F_1, \ldots, \dim F_k \}$. For many years, efforts to characterize the numbers of faces of general convex polytopes have focused on the more general question of determining all such flag numbers. One form this takes is the study of the cd-index, a bivariate noncommutative polynomial that encapsulates the information in the set of flag numbers.

We will give an overview of the state of research on this question, which has led in a number of unexpected directions. It gives further evidence of the strong influence of algebra in combinatorics and geometry. No previous background will be assumed.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**




The OLIVER CLUB

presents

**Harry Kesten**

of

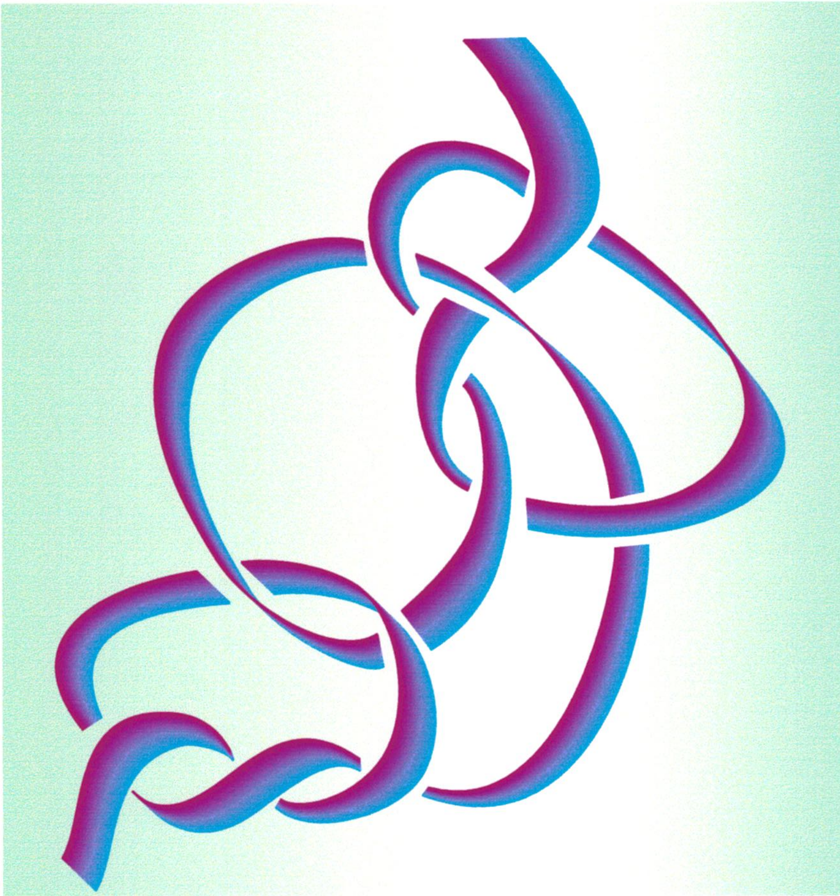
Cornell University

Thursday, October 1
4:15 p.m.
328 White Hall
Cornell University

# Percolation of arbitrary words in $\{0,1\}^{\mathbb{N}}$

Let $\mathcal{G}$ be a (possibly directed) locally finite graph with countably infinite vertex set; the most important example is $\mathcal{G} = \mathbb{Z}^d$. Assign independently to each vertex $v$ a 0 or a 1, with probability $p$ or $1 - p$ respectively. We want to know which sequences of zeroes and ones occur with positive probability along some self-avoiding path on $\mathcal{G}$. The traditional problem in site percolation is whether the sequence *(1,1,1, ... )* occurs on some path starting at a fixed vertex $v_0$. So-called AB-percolation occurs if the sequence *(1,0,1,0,1,0, ... )* occurs with positive probability on some path starting at $v_0$. We concentrate here on the questions (a) whether (with positive probability) all words are seen from $v_0$ and (b) whether all words are seen somewhere on $\mathcal{G}$ with probability 1. We also consider similar questions with ``all words" replaced by ``almost all words".

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB

presents

# John Conway

of
Princeton University

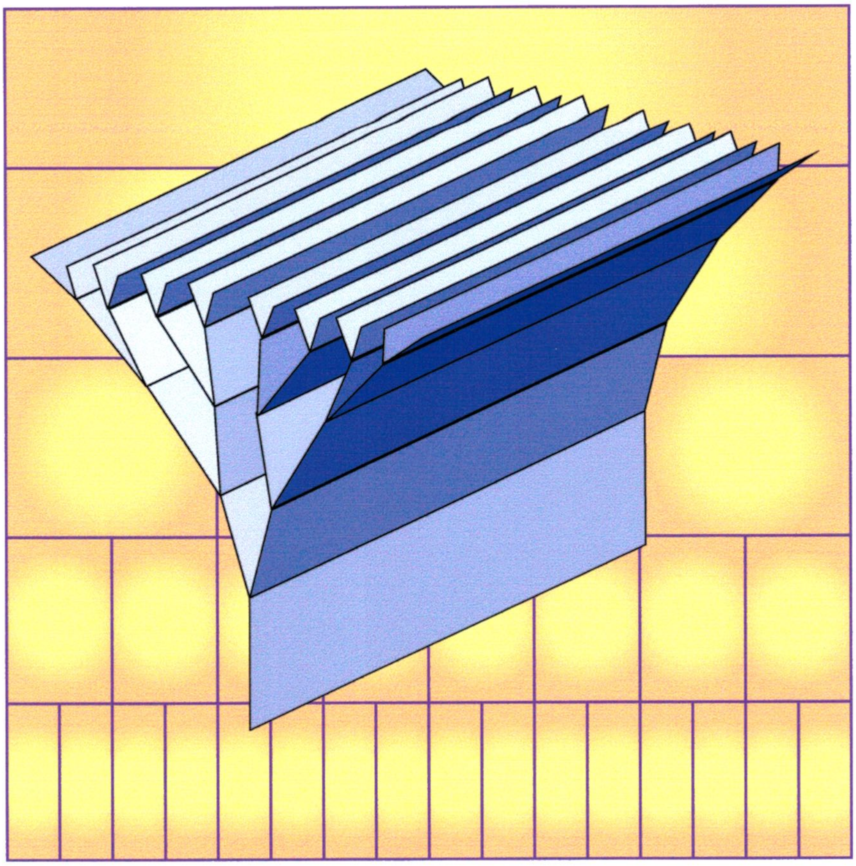
Thursday, October 8
4:15 p.m.
328 White Hall
Cornell University

# Tangles, Bangles and Knots


*Abstract*

How can you remember the shape of the knot your garden hose got into last winter? Or how would you describe it over the 'phone to someone else? I'll show how lots of tangles can be described by arithmetical fractions, and how these tangles can be put together to "explain" lots of knots.


**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB

presents

**Benson Farb**

*of the*

University of Chicago

Thursday, October 15
4:15 p.m.
328 White Hall
Cornell University

# The asymptotic geometry of groups


*Abstract*

Every finitely-generated group can be viewed as a metric space in a way which is unique up to maps of bounded distortion. In this talk I will discuss the recent discovery that many finitely-generated groups are actually determined uniquely by their asymptotic geometry. I will also describe how this phenomenon relates to the Mostow Rigidity Theorem, Gromov's Polynomial Growth Theorem, and Dynamical Systems.

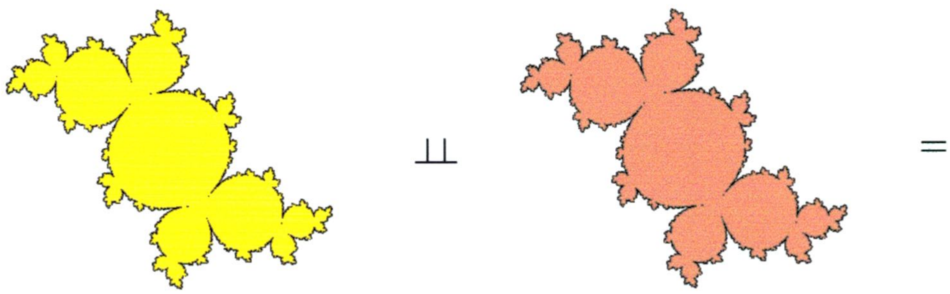

**Refreshments will be served at 3:45**
**in the Math Department Lounge**

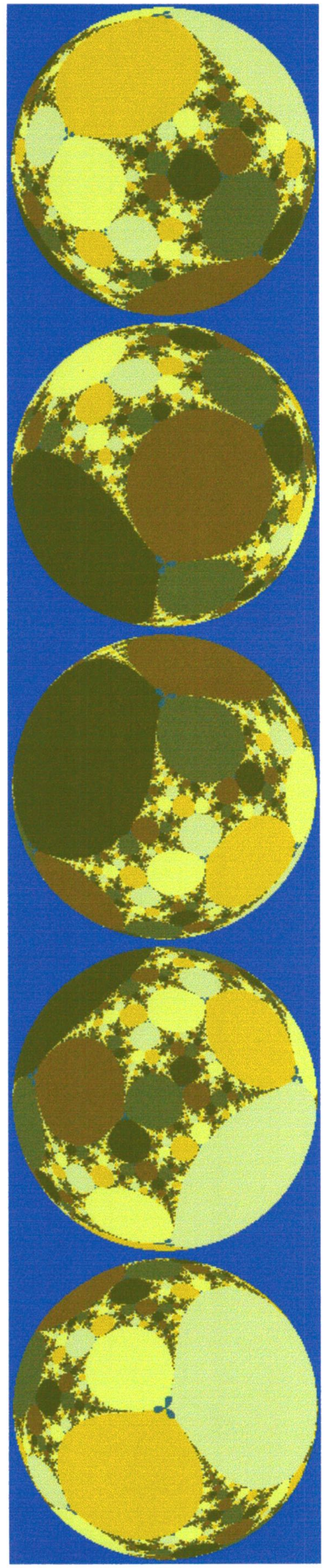
The 1998 Harry S. Kievel Lecture in Mathematics


# John W. Milnor

*Distinguished Professor*
*SUNY Stony Brook*


Thursday, October 22
4:30 p.m.
Hollis Cornell Auditorium
Goldwin Smith Hall
Cornell University Campus

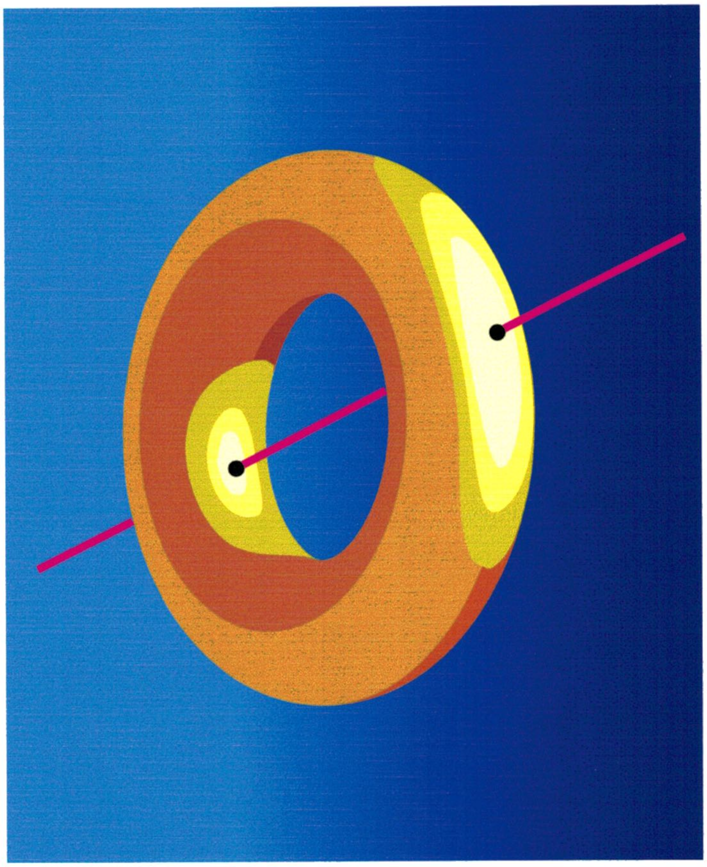




# Pasting together Julia sets


### *Abstract*

This talk will describe one surprising example which illustrates the "mating" construction, originated by Douady and Hubbard. By suitably pasting together two very complicated and skinny subsets of the plane one can reconstruct a smooth two-dimensional sphere.



The OLIVER CLUB

presents

**Maxim Braverman**

*of the*

Ohio State University

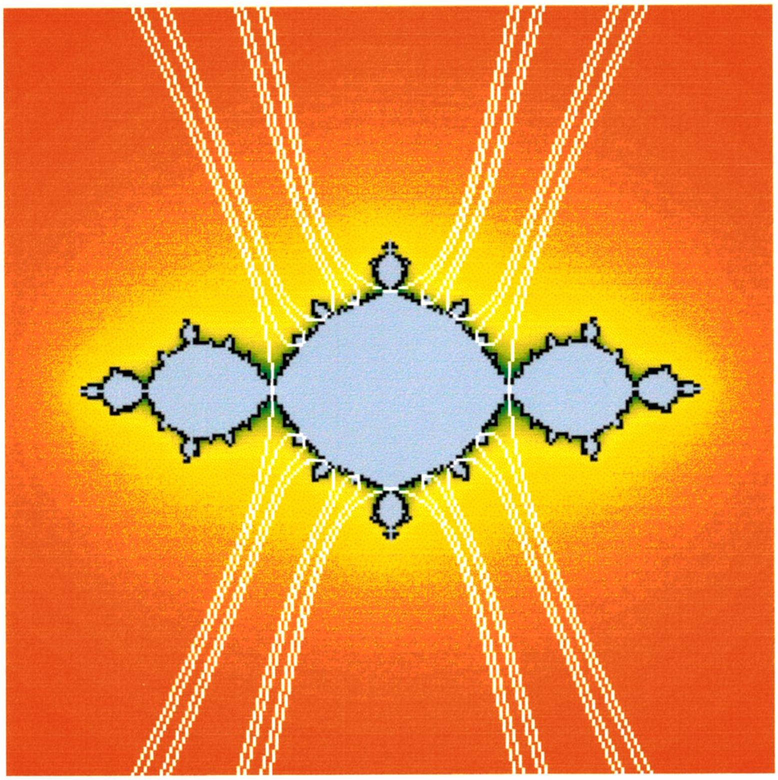
Thursday, October 29
4:15 p.m.
328 White Hall
Cornell University

# Morse theory for multi-valued functions


*Abstract*

Any smooth function on a compact manifold has at least 2 critical points: maximum and minimum. The celebrated Morse inequalities imply that on most manifolds each smooth function has many more critical points. For example, any function on the two-dimensional torus has at least 4 critical points (provided those points are non-degenerate).
In 1981, S. Novikov extended the Morse inequalities to multi-valued functions. In my talk, I'll review the Morse and Novikov theories and present a generalization of the Novikov inequalities to multi-valued functions with non-isolated critical points due to M. Farber and myself. If the time permits, I'll also discuss some applications.


**Refreshments will be served at 3:45
in the Math Department Lounge**



The OLIVER CLUB
presents

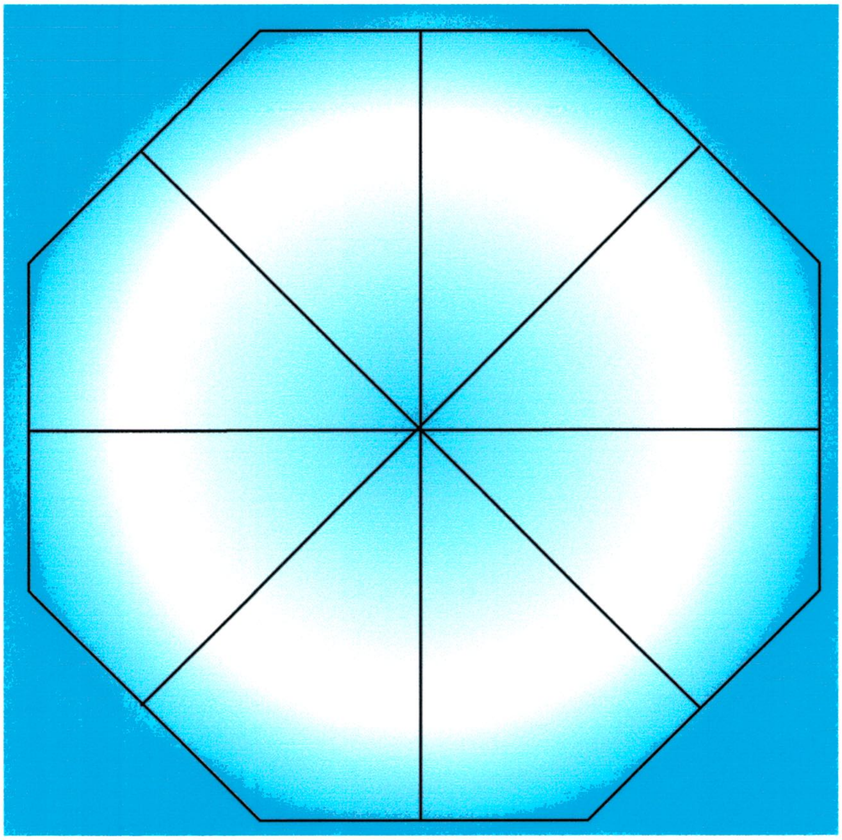
**Ricardo Perez-Marco**
*of*
U.C.L.A.

Thursday, November 5
4:15 p.m.
328 White Hall
Cornell University

# The butchered Riemann sphere and the new renormalization

In this talk we describe a new approach to renormalization of quadratic polynomials. It is based on new analytic tools that make sense of quotients of complex manifolds without fundamental domains. It is well known that by collapsing into points the intervals in the exterior of the triadic Cantor set we get a topological space homeomorphic to an interval. But classically no natural smooth structure is inherited by this topological quotient. In our dynamical problem, the Riemann sphere presents an infinite number of holes, dense and of total measure (the "butchered Riemann sphere"). We have a well defined topological quotient that collapses each of these holes into a Jordan arc. We prove that there is a natural complex structure in the quotient for which these Jordan arcs are analytic. The quotient mapping is a correspondence of the Riemann sphere that can be though of as a solution of a degenerate Beltrami equation.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

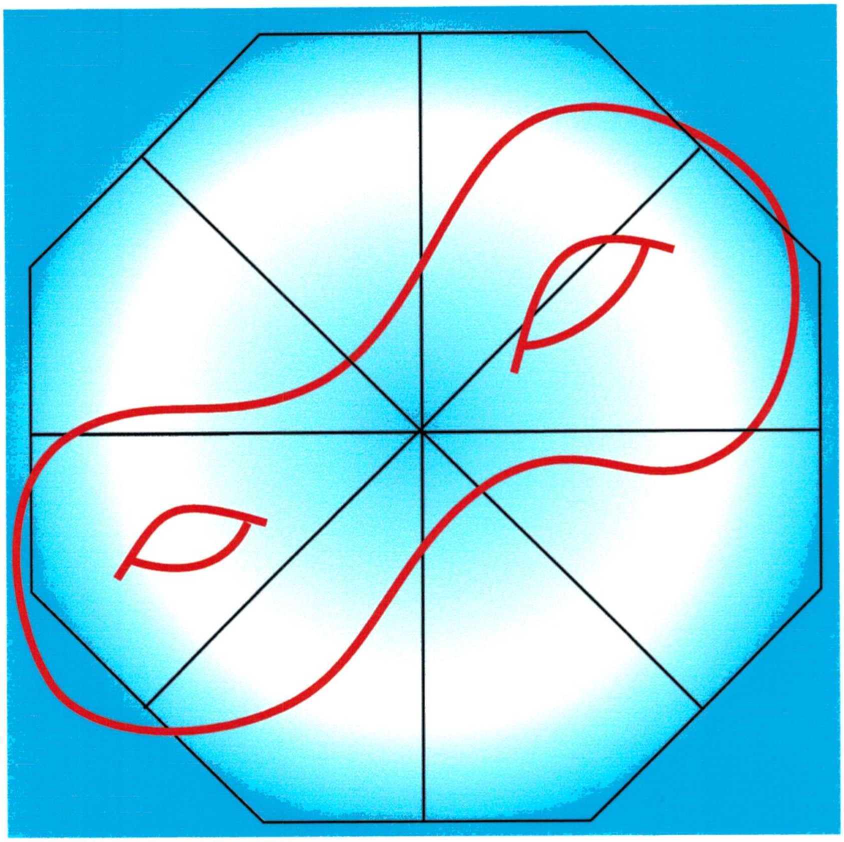
**Tom Farrell**
*of*
S.U.N.Y Binghamton

Thursday, November 12
4:15 p.m.
328 White Hall
Cornell University

# Topological rigidity

It is a classical theorem in topology that a closed manifold whose universal cover is contractible is determined up to homotopy equivalence by its fundamental group. The Borel conjecture is that in fact the fundamental group determines such a manifold up to homeomorphism. I will discuss joint work with Lowell Jones, in which we make progress towards Borel's conjecture, proving it under additional hypotheses on curvature and dimensions. Some applications will be given, including an application to harmonic maps which represents joint work also with Pedro Ontaneda.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

**Tom Farrell**
*of*
S.U.N.Y Binghamton

Thursday, November 12
4:15 p.m.
328 White Hall
Cornell University

# Topological rigidity

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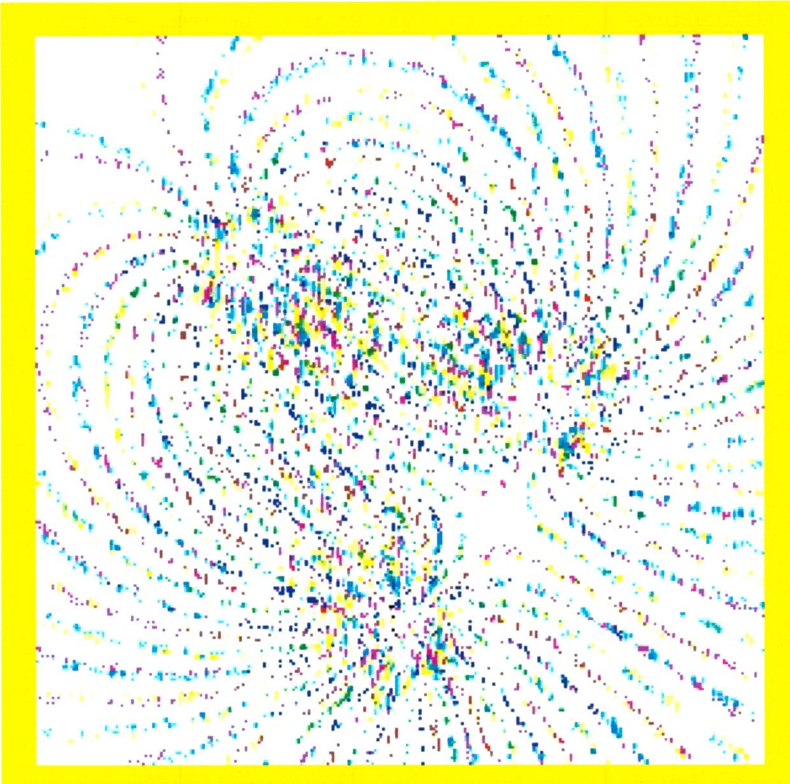
**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

**Dexter Kozen**
*of*
*Cornell University*

Thursday, November 19
4:15 p.m.
328 White Hall
Cornell University

# Kleene algebra with tests

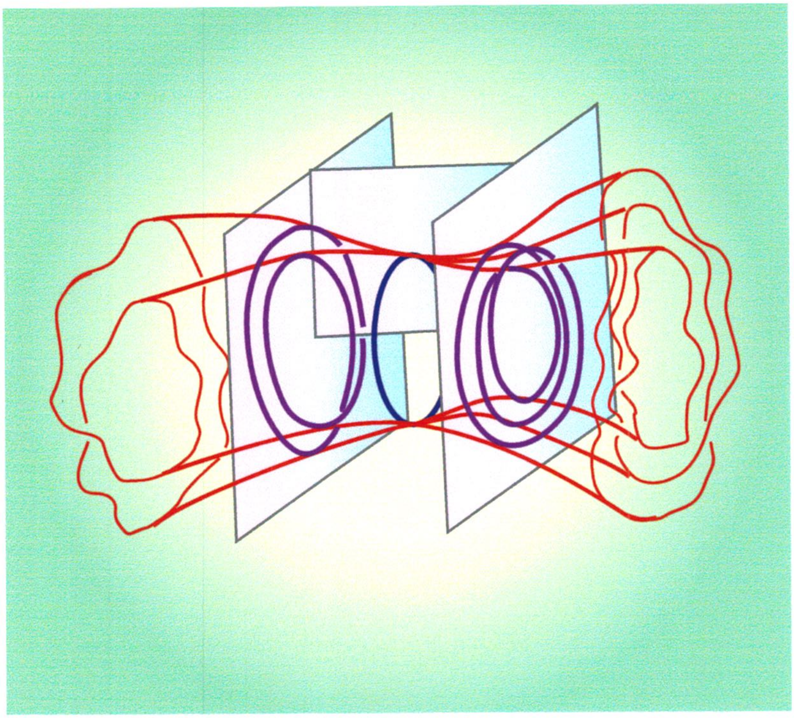
Kleene algebra is the algebra of regular expressions. It arises in many contexts: relational algebra, semantics and logics of programs, automata and formal language theory, and the design and analysis of algorithms.

A Kleene algebra with tests is a Kleene algebra with an embedded Boolean subalgebra. The presence of tests allows us to model conventional programming constructs such as conditionals and while loops. Many low level program manipulations such as loop unwinding and basic safety analysis can be carried out purely equationally in this system.

I will illustrate the use of Kleene algebra with tests by giving an equational proof of the following folklore result: every while program can be simulated by a while program with at most one while loop. This is interesting because of the widely held belief that there is no purely propositional proof of this result, and that it cannot be achieved without introducing extra variables.

I will also discuss completeness, complexity, and expressiveness results.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

# Yulij Ilyashenko

*of*
*Cornell University*

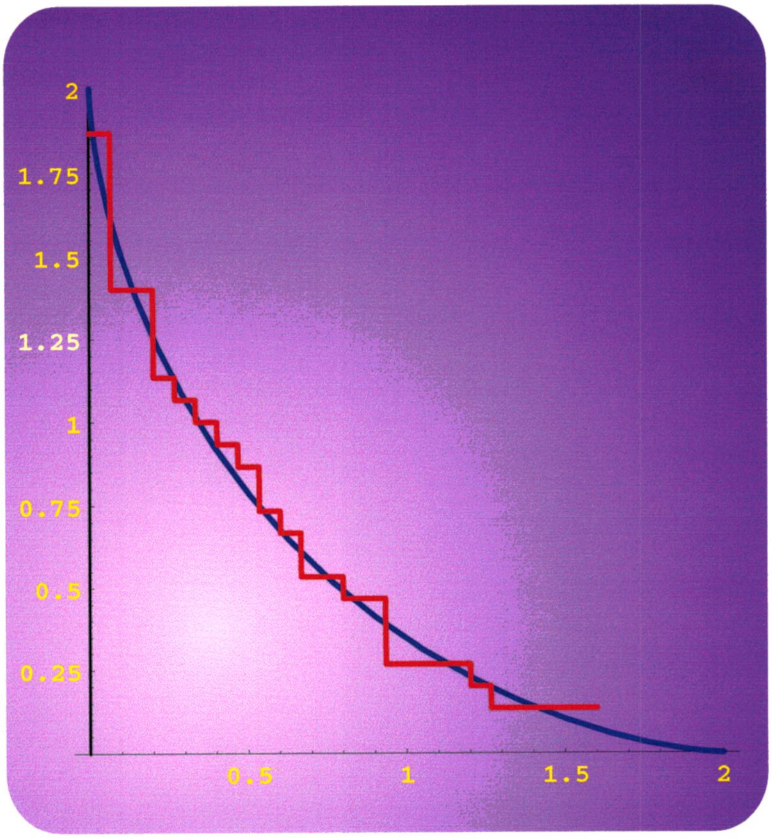
Thursday, December 3
4:15 p.m.
328 White Hall
Cornell University

# Non-local bifurcations and random dynamical systems

A simple example of a random dynamical system is the following. Take a random sequence of zeroes and ones and two diffeomorphisms of a circle: $f_0$ and $f_1$. Construct a product $F_n$ of $n$ factors, each one equal to $f_0$ or $f_1$, in the following way: the $k$-th factor is $f_0$ iff the $k$-th element of the random sequence is zero; otherwise, it is $f_1$. The sequence of these products $F_n$ is a random dynamical system on the circle.

Random dynamical systems seem to be quite different from classical ones, namely, from iterates of diffeomorphisms or phase flows generated by vector fields. Yet it occurs that random dynamical systems appear as subsystems of classical ones. Examples are provided by the theory of nonlocal bifurcations. The latter theory studies sudden changes in global geometric properties of parameter-dependent dynamical systems that occur when the parameter passes through a so called critical value. Nonlocal bifurcations that produce invariant sets with random dynamical systems on them, together with some other wonderful bifurcations, will be described in the talk.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**




The OLIVER CLUB

presents

**Jon A. Wellner**

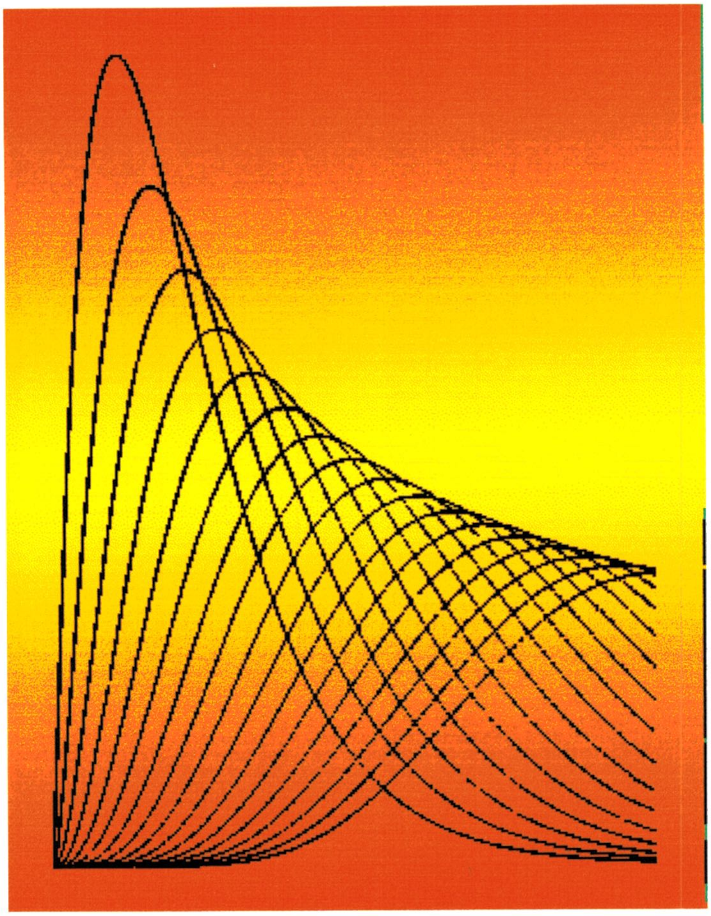
University of Washington

Monday, Jan. 25
4:00 p.m.
328 White Hall

# Longest increasing subsequences in random permutations

Let $L_n$ denote the length of the longest increasing subsequence of a random permutation of the integers $\{1, \ldots, n\}$. It was shown in 1972 that $L_n/\sqrt{n}$ approaches a constant as $n$ goes to infinity. In 1977 the value of the limiting constant was determined to be 2, in agreement with computer simulations performed in 1968 for values of $n$ up to $10^4$. The natural next set of questions concern the variance of $L_n$; how fast does $\mathrm{Var}(L_n)$ grow? In this talk I will discuss what is currently known, present some experimental (statistical!) evidence, and make some conjectures about the precise nature of the asymptotic behavior of $L_n$.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

**Michael Nussbaum**
*of*
Weierstrass Institute
Berlin

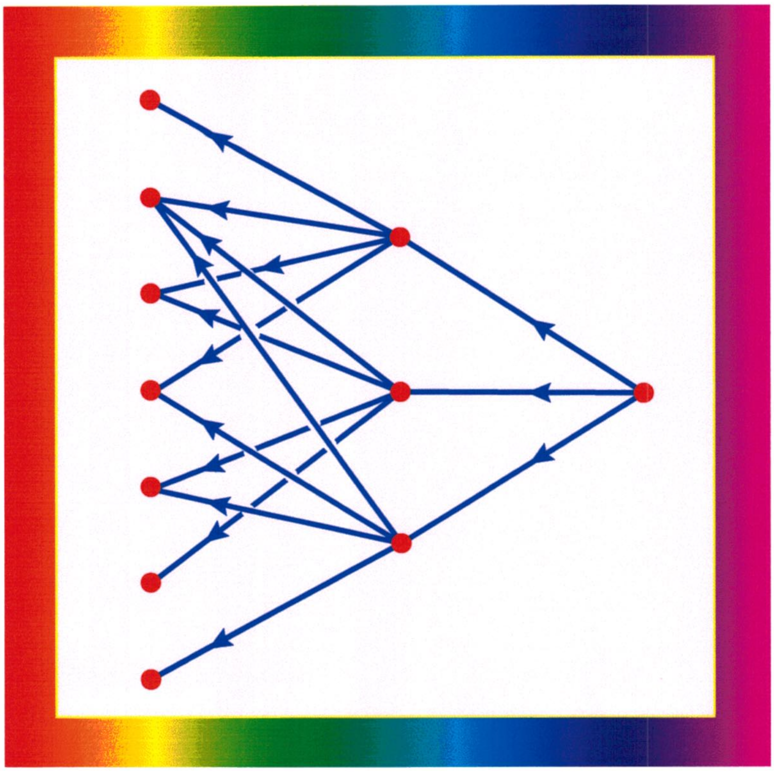
Thursday,February 4
4:15 p.m.
328 White Hall
Cornell University

# Asymptotic equivalence of statistical experiments

An experiment is a family of probability measures; it is the basic object in mathematical statistics. In the Gaussian case, optimal decisions can be found, but frequently there is only a central limit theorem stating weak convergence. For optimal decisions to carry over, one would need strong or total variation convergence. In this connection the concept of deficiency approximation has been developed (Le Cam theory).

We discuss recent developments for infinite dimensional experiments, connected with stochastic ill-posed inverse problems.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

**Dan Rockmore**

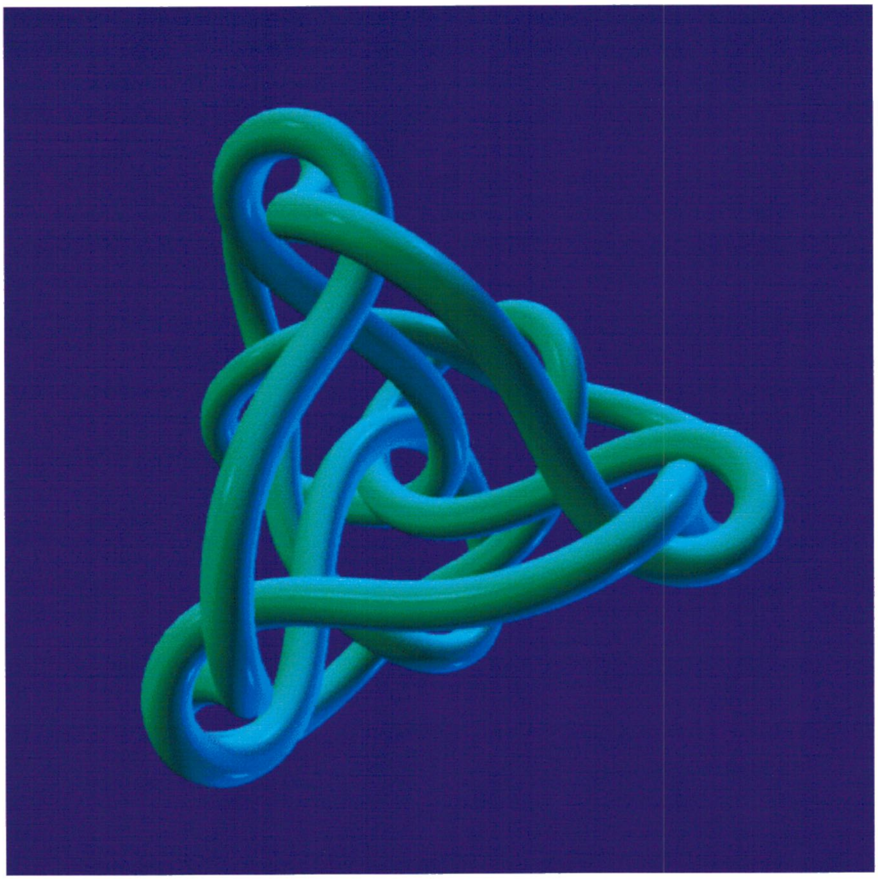
*of*
Dartmouth College

Thursday, February 11
4:15 p.m.
328 White Hall
Cornell University

# Conjectures, applications and algorithms for Fourier analysis on $SL_2(F_p)$

The ability to explicitly compute the Fourier transform of a function on $SL_2(F)$, for F a finite field, has (1) produced a number of conjectures regarding the spectrum of various Cayley graphs on these groups and related questions in quantum chaos and (2) had applications to fast encoding algorithms for expander codes. In this talk I will survey these conjectures and applications and discuss the efficient algorithms which make them possible.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

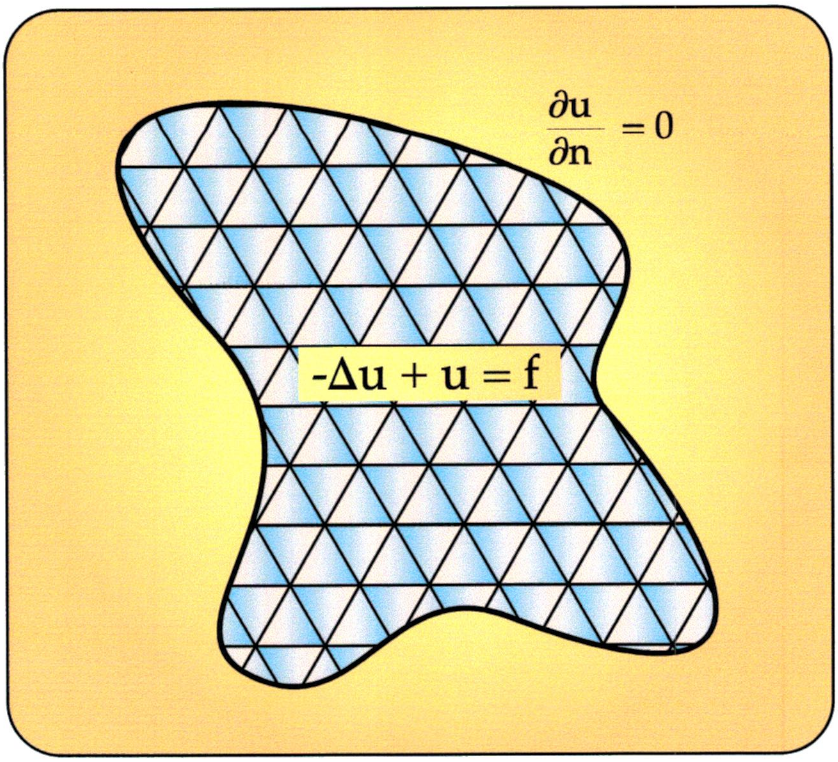
**Allen Hatcher**
*of*
Cornell University

Thursday, February 18
4:15 p.m.
328 White Hall
Cornell University

# Knots and symmetry

It is a long-standing open question whether there exists an 'energy function' on the space of all smooth knots in 3-space, whose gradient flow gives a continuous deformation of every knot onto a minimal-energy configuration for the knot. In particular, such a minimal-energy configuration should exhibit any symmetries latent in the knot. As will be described in the talk, for many knot types there are at least no topological obstructions to the existence of such an energy flow, and the homotopy type of the space of all knots of the given knot type has a nice description in terms of the symmetry group of the knot.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**




The OLIVER CLUB
presents

# Al Schatz

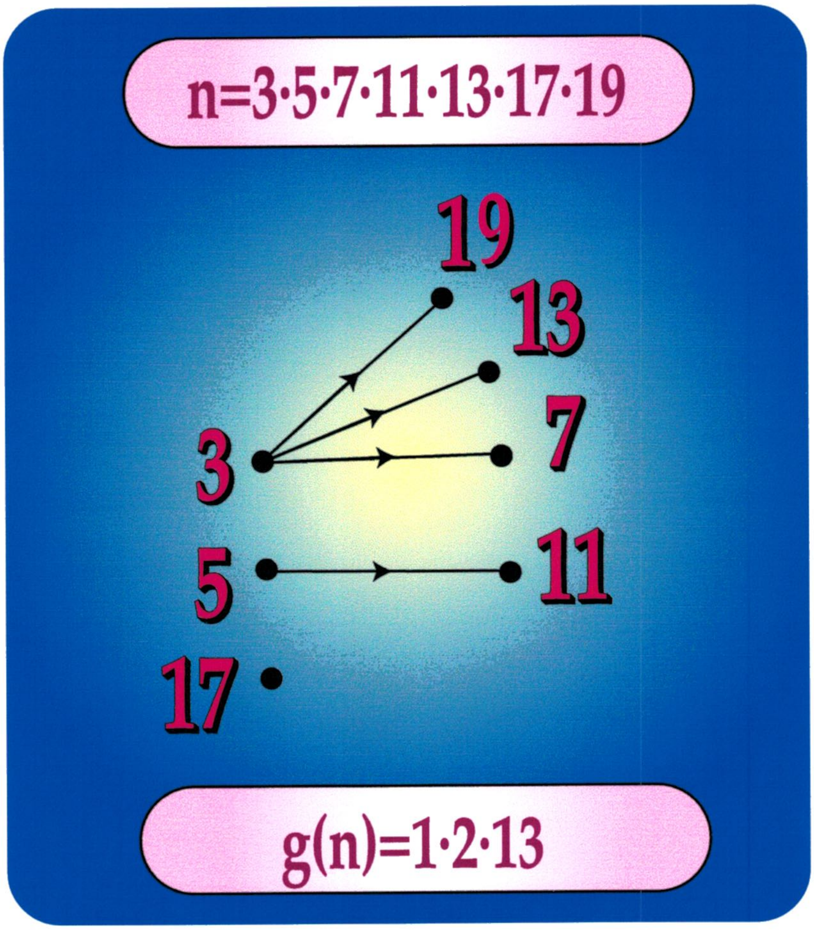
*of*
Cornell University

Thursday, February 25
4:15 p.m.
328 White Hall
Cornell University

# Errors in the finite element method

We shall consider the question of the accuracy of finite element approximations of solutions of second order elliptic boundary value problems. Our goal will be to discuss some recent a priori estimates which describe the structure of the pointwise error. An attempt will be made to make the talk self-contained by summarizing the necessary background in partial differential equations and the finite element method.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**




The OLIVER CLUB

presents

**Keith Dennis**

*of*

Cornell University

Thursday, March 4

4:15 p.m.

328 White Hall

Cornell University

# The number of groups of order *n*

Let $g(n)$ be the number of groups of order $n$. This function is essentially impossible to compute and number-theoretic investigations of its density properties are difficult. Nevertheless, one can ask:

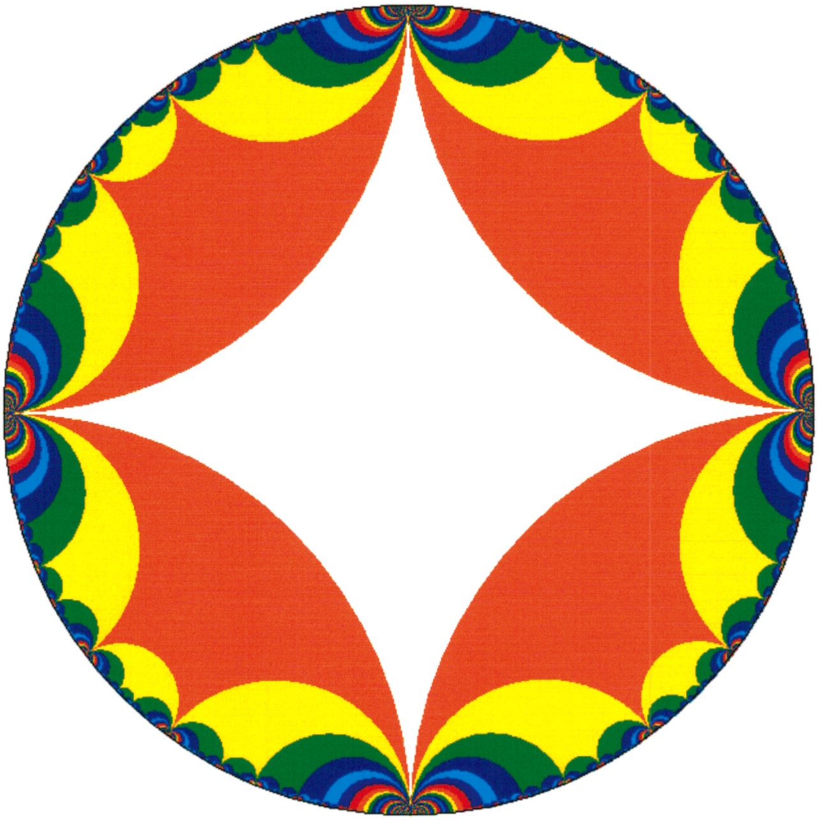
"What values does $g(n)$ attain?"

The path of attack will be

Algebra ----> Geometry ----> Number Theory ----> Analysis

For special values of $n$ one can construct a labelled, directed graph, from which $g(n)$ can be computed easily via geometric properties of the graph. A further restriction to the case of rooted trees converts the function "$g$" into a homomorphism. The numbers that arise for the trees turn out to be natural generalizations of Fibonacci numbers. All of the information for "$g$" on trees can then be combined into a Zeta function.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

# Jim Cogdell

*of*
Oklahoma State
University

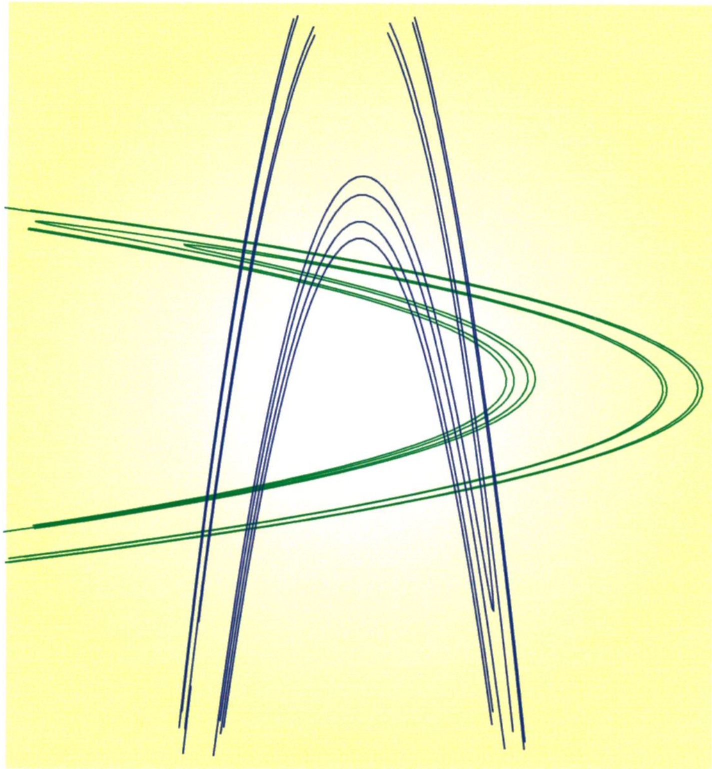
Thursday, March 11
4:15 p.m.
328 White Hall
Cornell University

# Arithmetic Dirichlet series and automorphic forms

Ever since Riemann, number theorists have found it fruitful to attach a complex analytic invariant, the L-function, to arithmetic objects, such as Dirichlet characters, elliptic curves, etc.. These L-functions, or Arithmetic Dirichlet Series, enjoy several nice properties (Euler product, duality, a convolution structure) and are conjectured to enjoy others (analytic continuation and functional equation). On the other hand, there are natural analytic objects, namely modular or automorphic forms, which have L-functions that also enjoy these properties, including the continuation and functional equation. Conjecturally, all the Arithmetic Dirichlet Series come from automorphic forms. For the invariants attached to elliptic curves this was (essentially) demonstrated by Wiles.

We hope to give a down to earth exposition of these Arithmetic Dirichlet Series and explain to what extent their nice properties actually characterize those invariants coming from automorphic forms.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

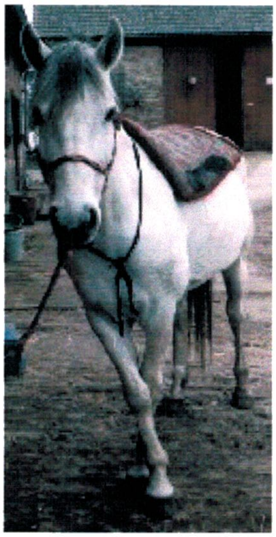
# John Smillie

*of*
Cornell University

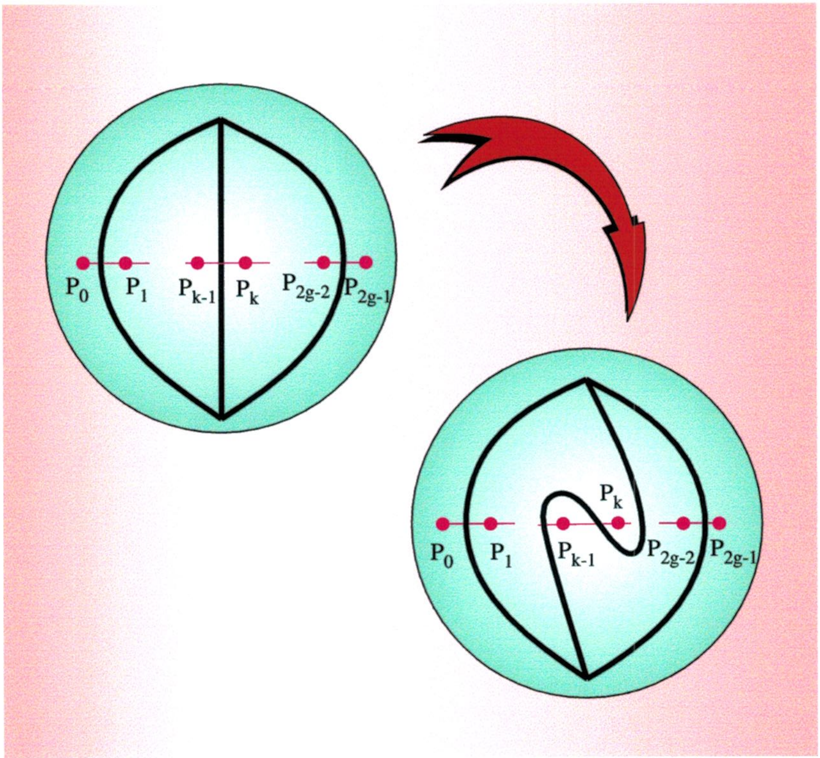
Thursday, March 18
4:15 p.m.
328 White Hall
Cornell University

# When is a horseshoe not a horseshoe?

Prior to the 1960's chaotic behavior in dynamical systems was quite mysterious. In the 1960's Smale described his horseshoe example which provided a simple model of chaotic behavior. This example and others led to the theory of hyperbolic dynamical systems. This theory answers many general dynamical questions but still leaves some concrete questions about explicit diffeomorphisms unanswered. We will look at a natural two parameter family of diffeomorphisms of $\mathbf{R}^2$ where Smale's horseshoe arises. Using some unorthodox techniques we will show how the horseshoe property is lost as the parameters vary.



**Refreshments will be served at 3:45**
**in the Math Department Lounge**




The OLIVER CLUB
presents

**Yurii M. Burman**

*of*

Independent University of Moscow

Thursday, April 1
4:15 p.m.
328 White Hall
Cornell University

# Trees, quadrangulations, and moduli spaces

Let $M$ and $N$ be even-dimensional oriented real manifolds, and $u : M \dashrightarrow N$ be a smooth mapping. A pair of complex structures at M and N is called *u-compatible* if the mapping $u$ is holomorphic with respect to these structures. The quotient of the space of $u$-compatible pairs of complex structures by the group of $u$-equivariant pairs of diffeomorphisms of $M$ and $N$ is called a moduli space of $u$-equivariant complex structures. The talk is devoted to a description of the fundamental group $G$ of this moduli space in the following case: $N = CP^1$, $M \subset CP^2$ is a hyperelliptic genus $g$ curve given by the equation $y^2 = Q(x)$ where $Q$ is a generic polynomial of degree $2g+1$, and $u(x,y) = y^2$. The group $G$ is a kernel of several (equivalent) actions of the braid-cyclic group $BC_{2g}$ on $2g$ strands. These are: an action on the set of trees with $2g$ numbered edges, an action on the set of all splittings of a $4g+2$-gon into numbered nonintersecting quadrangles, and an action on a certain set of subgroups of the free group with $2g$ generators. $G_{2g} < BC_{2g}$ is a subgroup of index $(2g+1)^{2g-2}$.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**


The OLIVER CLUB
presents

**Joel Spencer**

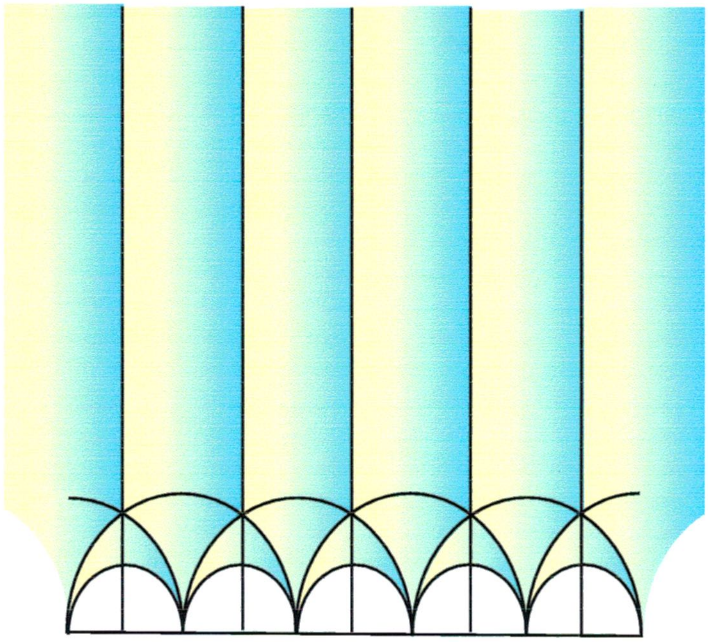
Courant Institute

Thursday, April 8
4:15 p.m.
328 White Hall
Cornell University

# The strange logic of random graphs

Paul Erdös and Alfred Rényi were the first to consider the evolution of the random graph $G(n,p)$ ($n$ vertices, edge probability $p$) as $p=p(n)$ evolved from zero to one and saw that for many natural properties $A$ there was a threshold function (today we might refer to a percolation) near which the probability of $A$ went quickly from near zero to near one. For example, when $p = 0.99(\ln n)/n$ the graph is almost surely disconnected while when $p = 1.01(\ln n)/n$ the graph is almost surely connected. Their threshold functions were always of the form $p(n) = n^{-\alpha+o(1)}$ with $\alpha$ rational. We blend logic to this brew, making statements for any first order property $A$. For $\alpha$ irrational with Saharon Shelah we have a Zero-One Law, that for any such $A$ the limiting probability is either zero or one. This leads to a complete theory and then to countable graphs that model the asymptotic properties in an intriguing though nondirect way. These graphs (and their automorphism groups) are quite interesting by themselves.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
presents

# E. B. Vinberg

Moscow State University

Thursday, April 15
4:15 p.m.
328 White Hall
Cornell University

# The first Betti number of hyperbolic manifolds and generalizations

Let $X$ be an irreducible symmetric space of non-positive curvature and $M$ a compact manifold locally isometric to $X$. If rk $X>1$ then it follows from results of D. Kazhdan and L. Vasserstein that the first Betti number of $M$ equals 0. In contrary with this, if $X$ is the $n$-dimensional Lobachevsky space, there is a Thurston conjecture that $M$ admits a finite covering whose first Betti number is positive. It is trivial for $n$=2 and is proved in many cases for $n$>2. Recently A. Lubotzky proved that, more generally, in many cases a compact hyperbolic manifold $M$ admits a finite covering whose fundamental group maps onto a free group of rank 2. Margulis and the speaker generalized his results and simplified the proofs. In particular, the following result is obtained: any non-affine infinite irreducible Coxeter group has a finite index subgroup mapping onto a free group of rank 2.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**


**The OLIVER CLUB**
presents

# Alice Chang

*of*
Princeton University

Thursday, April 22
4:15 p.m.
328 White Hall
Cornell University

# On the Chern-Gauss Bonnet Integral on 4-manifolds

When $M$ is a complete surface of finite total curvature, A classical result of Cohn-Vossen showed that if the Gauss curvature $K$ of an analytic metric is absolutely integrable then $\int KdA \leq 2\pi\chi$, where $\chi$ is the Euler number of the surface. The inequality was later extended by Huber to metrics with weaker regularity; he also proved that such a surface can be conformally compactified by adjoining a finite number of points. In this talk, I will discuss a generalization of the above results to a class of complete, locally conformally flat 4-manifolds, where we replace the Gauss curvature by a variation of the Chern-Gauss-Bonnet integrand on 4-manifolds. I will discuss how to relate the integrand to a fourth-order PDE and how to relate the boundary behaviour of the PDE to study the geometry problem. This is a report of some recent joint work with Jie Qing and Paul Yang.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**

The OLIVER CLUB
presents

# Henri Darmon

*of*
McGill University

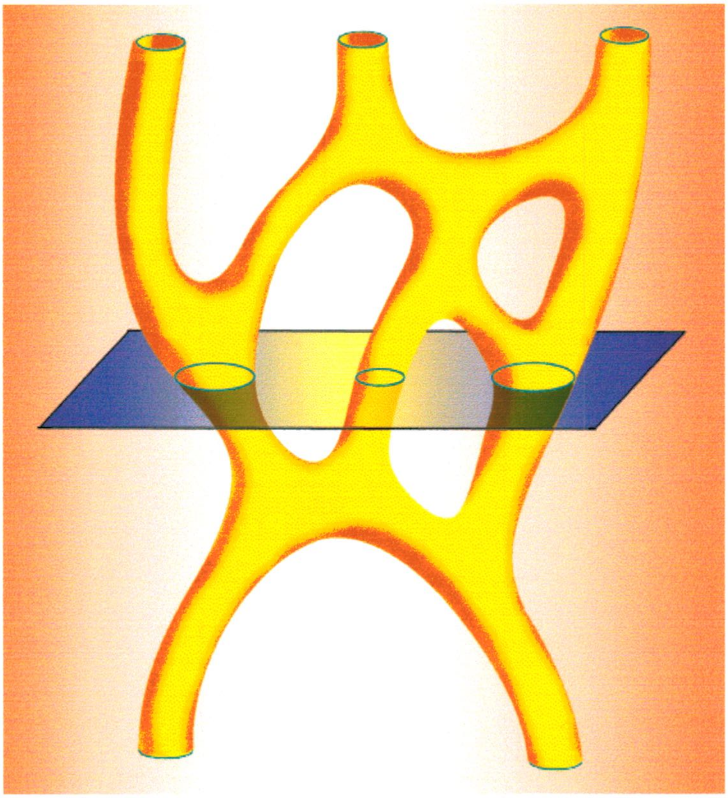
Thursday, April 29
4:15 p.m.
328 White Hall
Cornell University

# Fermat est mort, vive Fermat!


*ABSTRACT*

After Wiles' proof of Fermat's Last Theorem, a number theorist can find solace in the study of the *generalized Fermat equation* $x^p + y^q = z^r$. I will explain the connection of this equation with Hecke triangle groups, hypergeometric equations, and a generalization of the Shimura-Taniyama conjecture for abelian varieties with real multiplications.


**Refreshments will be served at 3:45**
**in the Math Department Lounge**



The OLIVER CLUB
and the
CORNELL TOPOLOGY
FESTIVAL
present

# Yakov Eliashberg

*of*
Stanford University

Friday May 7
4:30 p.m.
Kaufmann Auditorium
Cornell University

# Introduction to symplectic field theory

*ABSTRACT*

Symplectic field theory is a new theory (currently under joint construction by A. Givental, H. Hofer and the author), whose goal is to provide, on the one hand, new invariants of contact manifolds and Legendrian knots in them and, on the other hand, to give a way of computing Gromov-Witten invariants of symplectic manifolds and their Lagrangian submanifolds by cutting them into elementary pieces.

**Refreshments will be served at 3:45**
**in the Math Department Lounge**