MATH 275 - Final Practice Solutions

Chapter 1 Section 1.6.

4. $P[R > G] = \frac{1}{2}$

8. **(a)** $P = \frac{1}{2} + \frac{1}{4}$; **(b)** $P = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}$

16. $16\% + 8\% + 53\% = 77\%$

44. $\frac{\binom{5}{2}\binom{5}{2}}{\binom{10}{4}}$

52. $\frac{\binom{10}{2}+\binom{8}{2}+\binom{6}{2}}{\binom{24}{2}}$

60. $\frac{20\cdot 19\cdot 18\cdot 17\cdot 16}{20^5}$

61. **(a)**$(\frac{1}{2})^5(\frac{1}{2})^3$; **(b)**$\frac{\binom{5}{5}}{\binom{8}{5}}$

73. $5(\frac{4}{5})^{10} - 10(\frac{3}{10})^{10} \leq P \leq 5(\frac{4}{5})^{10}$

Chapter 2 Section 2.5.

7. **(a)**$P(A \cup B) = 0.92$; **(b)**$P(\text{only}A \text{ or only}B) = 0.44$

12. $P[N = 0] = (0.1)(\frac{1}{9})(0.25)$; $P[N = 1] = (0.9)(\frac{1}{9})(0.25) + (0.1)(\frac{8}{9})(0.25) + (0.1)(\frac{1}{9})(0.75)$;
, $P[N = 2] = (0.9)(\frac{8}{9})(0.25) + (0.1)(\frac{8}{9})(0.75) + (0.9)(\frac{1}{9})(0.75)$; $P[N = 3] = (0.9)(\frac{8}{9})(0.75)$

18. **(a)**$\binom{20}{2}(0.108)^2(0.892)^{18}$; **(b)**$(0.892)20 + 20(0.108)(0.892)^{19} + 190(0.108)^2(0.892)^{18}$

21. **(a)**$\binom{10}{8}(\frac{1}{2})^{10} + \binom{10}{9}(\frac{1}{2})^{10} + (\frac{1}{2})^{10}$; **(b)**$\binom{10}{8}(0.9)^8(0.1)^2 + \binom{10}{9}(0.9)^9(0.1) + (0.9)^{10}$

25. $P[\text{at least } 1H] = 0.75,\ P[HT \text{ or } HH] = 0.5,\ P = \frac{0.5}{0.75}$

31. $P[\text{reads at least } 1] = 70,\ P(A) = 60,\ P = \frac{0.6}{0.7}$

40. $P = 1 - P[d \leq 3] = 1 - \frac{9}{\binom{10}{2}} - \frac{8}{\binom{10}{2}} - \frac{7}{\binom{10}{2}}$

43. $\frac{1}{6}$

59. $P(B|T^+) = \frac{1/1000}{1/1000+(999/1000)(0.05)}$

63. $P(B|\text{say } B) = \frac{(0.80)(0.15)}{(0.80)(0.15)+(0.20)(0.85)}$

76. $P(\text{bag } 1|G) = \frac{1/3}{1/3+(1/3)(0.5)} = \frac{2}{3}$

Chapter 3 Section 3.8.

6. $1 - (\frac{1}{2})^n = 0.99,\ n = \log_{1/2}(0.01)$

7. **(a)** $\frac{\binom{k-1}{2}}{\binom{15}{3}}$; **(b)** 15

17. $P[X \leq 3] = e^{-3}[1 + 3 + \frac{3^2}{2}]$

21. $P = 1 - [1 - \frac{e^{-1}}{5!}]^{300}$

25. $E[\text{gain}] = \frac{1}{2} - (\frac{5}{6})^3$

27. $E[\text{gain}] = -\frac{2}{38}$; **(b)** $E[\text{gain}] = \frac{8}{38}$; **(c)** $-\frac{2}{38}$

33. $(\frac{1}{0.6} - 1)4 + 20$

38. **(a)** $\frac{.15}{.05+.15+.20}$; **(b)** $\frac{.2}{.05+.1+.2}$

41. the two numbers can be $(\frac{5}{9} + \frac{1}{3}, \frac{5}{9} - \frac{1}{3})$ or $(\frac{5}{9} - \frac{1}{3}, \frac{5}{9} + \frac{1}{3})$

44. $EX = \frac{\binom{15}{2}\binom{10}{2}+2\binom{15}{1}\binom{10}{2}+3\binom{10}{3}}{\binom{25}{3}}$; $E(X - Y) = 2EX - 3$

45. **(a)** $12[1 - (\frac{1}{6}(0.6))^{12}]$; **(b)** $6(0.6)$

50. $EX = \frac{11}{3},\ VarX = 15 - (\frac{11}{3})^2$

60. $EX = 0,\ EY = EX^2 = 2,\ Cov(X, Y) = EX^3 - EXEX^2 = 0$

Chapter 4 Section 4.5.

1. $P(X \geq 75) \leq \frac{EX}{75} = \frac{3}{4}$; **(b)** $P(40 < X < 60) \geq 1 - \frac{VarX}{100} = \frac{3}{4}$

2. $P(|X-2| \geq 2) \leq 1 - \frac{VarX}{4} = \frac{3}{4}$; $P(|X-2| \geq 2) = P(X=4) = (\frac{1}{2})^4$

8. $P(S_n \geq 45) = P(Z \geq 0.83) = 0.203$

10. $P(S_n \geq 23) = P(Z \geq 1.5) = 0.066$

14. $P(S_n \geq 2.7 \cdot 10^6) = P(Z \geq 3.775) = 0.001$

19. $P(X > 120) = P(Z > 2) = 0.1328$

25. P[serve at least 49 customers in 4 hrs] $= P$[49 customers take at most 4 hrs] $=$
, $= P[S_{49} \leq 240] = P[Z \leq \frac{240-245}{\sqrt{98}}] = 0.31$

28. $P[|\bar{X}_n - p| \geq 0.05] = P[|Z| \geq \frac{0.05}{\sqrt{p(1-p)/n}}]0.05$, $n = (\frac{1.96}{0.05})^2 p(1-p) \leq (\frac{1.96}{0.05})^2 \frac{1}{4} = 384$

Chapter 5 Section 5.7

2. X_t=color of the urn we draw from at time t, **(a)**$P = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} \\ \frac{1}{5} & \frac{4}{5} \end{bmatrix}$; **(b)**$p^3(R,B) = \frac{2}{5}\frac{3}{5} + \frac{3}{5}\frac{4}{5}$

6. **(a)** $p^3(1,4) = (0.4)^3$; **(b)** $p^3(1,0) = 0.6 + (0.4)(0.6)^2$

7. **(a)** $P = \begin{bmatrix} 0.4 & 0.6 \\ 0.1 & 0.9 \end{bmatrix}$;
(b) $p^2(r,r) = (0.4)^2 + (0.6)(0.1)$, $p^3(r,r) = (0.4)^3 + (0.4)(0.6)(0.1) + (0.6)(0.1)(0.4) + (0.6)(0.9)(0.1)$

8. $P_1(c) = 63.224\%$, $P_2(c) = 67.728\%$

10. $P_1(1) = 870$, $P_1(2) = 1150$, $P_1(3) = 980$; $P_2(1) = 782$, $P_2(2) = 1255$, $P_2(3) = 963$

12. **(a)** model predicts: 41.52% in 2000, 46.05% in 2010; **(b)** $\pi_h = \frac{2}{3}$

17. **(a)** $P = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$; **(b)** $\pi_w = \frac{3}{5}$

22. $\pi_w = 0.96$, $N = 30(0.96) = 28.8$