Martingales associated to minimal submanifolds and mean curvature flow

Rob Neel
Lehigh University

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Abstract

We introduce a class of martingales arising, for instance, from Brownian motion on minimal submanifolds, time-inhomogeneous "Brownian motion" along backward mean curvature flow, and the natural diffusion on some sub-Riemannian structures. We give applications to parabolicity and area growth for classical minimal surfaces, and to transience and the existence of non-constant bounded harmonic functions in the more general context of minimal submanifolds of Cartan–Hadamard manifolds. Time permitting, we describe work of Soner and Touzi showing how mean curvature flow can in fact be represented in terms of a target problem for such martingales, and we briefly mention current work extending these ideas to Ricci flow on surfaces.