Semimartingale reflecting Brownian motion in two dimensions: Exact asymptotics for the stationary distribution

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Abstract

We consider a two-dimensional semimartingale reflecting Brownian motion (SRBM) in the nonnegative quadrant. The data of the SRBM consists of a two-dimensional drift vector, a 2×2 positive definite covariance matrix, and a 2×2 reflection matrix. Assuming the SRBM is positive recurrent, we are interested in tail asymptotics of its marginal stationary distribution along each direction in the quadrant. For a given direction, the marginal tail distribution has the exact asymptotic of the form $bx^{\kappa} \exp(-\alpha x)$ as x goes to infinity, where α and b are positive constants and κ takes one of the values $-3/2$, $-1/2$, 0, or 1; both the decay rate α and the power κ can be computed explicitly from the given direction and the SRBM data.

A key tool in our proof is a relationship governing the moment generating function of the two-dimensional stationary distribution and two moment generating functions of the associated one-dimensional boundary measures. This relationship allows us to characterize the convergence domain of the two-dimensional moment generating function. For a given direction c, the line in this direction intersects the boundary of the convergence domain at one point, and that point uniquely determines the decay rate α. The one-dimensional moment generating function of the marginal distribution along direction c has a singularity at α. Using analytic extension in complex analysis, we characterize the precise nature of the singularity there. Using that characterization and complex inversion techniques, we obtain the exact asymptotic of the marginal tail distribution. Extensions to high-dimensional problems will also be discussed.

This talk is based on joint works with Masakio Miyazawa at Tokyo University of Science.