Gaussian free field, random measure and KPZ on $\mathbb{R}^4$

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Abstract

A highlight in the study of quantum physics was the work of Knizhnik, Polyakov and Zamolodchikov (1988), in which they proposed a relation (KPZ relation) between the scaling dimension of a statistical physics model in Euclidean geometry and its counterpart in the random geometry. Recently, Duplantier and Sheffield used the 2D Gaussian free field to construct the Liouville quantum gravity measure on a planar domain, and gave the first mathematically rigorous formulation and proof of the KPZ relation in that setting. We apply a similar approach to generalize part of their results and techniques to $\mathbb{R}^4$ (as well as to $\mathbb{R}^{2n}$ for $n \geq 2$). To be specific, we construct a random Borel measure on $\mathbb{R}^4$ which formally has the density (with respect to the Lebesgue measure) being the exponential of an instance of a 4D Gaussian free field. We also establish the KPZ relation corresponding to this random measure. This is joint work with Dmitry Jakobson.