

# Hypercontractivity, reverse hypercontractivity, and simple proofs

Ryan O'Donnell  
Carnegie Mellon University

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## Abstract

A random variable  $X$  is said to be " $(p, q, \rho)$ -hypercontractive" if some blah blah blah condition involving its moments holds. The practical upshot is that hypercontractive random variables behave very reasonably. Various simple random variables can be shown to be hypercontractive; furthermore, low-degree polynomials of independent hypercontractive random variables are also hypercontractive. These facts can be used to show some powerful theorems, like the "Kahn–Kalai–Linial Theorem" and the "Frankl–Rodl Theorem".

Luckily, all of these things can be proved fairly simply. That's good, because it means even computers can prove them. Applications of this fact to algorithmic optimization problems like graph bisection and chromatic number will be discussed.