Brownian rotations, and an infinite dimensional general position theorem

Todd Kemp
University of California, San Diego

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Abstract

Consider two random subspaces V and W of $\mathbb{C}^d$. Their intersection has dimension at least $\dim(V) + \dim(W) - d$; if there is equality (or 0 if $\dim(V) + \dim(W) < d$) then V and W are said to be in **general position**. If V and W are random subspaces, they are *almost surely* in general position. Depending on the definition of "almost sure," this theorem dates back 150 years or more. A more modern form could be stated thus: let U_t be a Brownian motion on the unitary group $U(d)$, independent from V and W; then, for any $t > 0$, $U_t(V)$ and W are in general position with probability 1.

What happens when the dimension d is infinite? While there is no unitarily invariant measure in infinite dimensions, it is still possible to make sense of the Brownian motion acting on *some* subspaces. However, the easy techniques for proving the general position theorem are unavailable.

In this talk, I will address my recent joint work with Benoit Collins in this infinite dimensional setting. Using the tools of random matrix theory and free probability, the question can be reduced to one of regularity for the solution of a certain family of complex PDEs. This allows us to prove not only the general position theorem, but also (a special case of) a long-standing conjecture about free entropy and information.