Project Based Math 112, Fall 2001
Integration Hints

1. If the integral isn't of a form you know how to do, try to use u-substitution or integration by parts to transform it into an integral you do know how to do.

2. Integration by Parts

   If the integral looks something like: $\int f(x)g(x)\ dx$,

   try integration by parts (especially if f and g are polynomials, trig functions, e^x, or $\ln x$). Remember that if you get $\int f(x)g(x)\ dx = h(x) - \int f(x)g(x)\ dx$, then you can solve algebraically for $\int f(x)g(x)\ dx$.

3. Trigonometric Integrals:

   (a) $\int (\sin^{2n+1}x) \cos^m x\ dx$ (odd power of sine)

   Rewrite as $\int \sin x\ (1-\cos^2 x)^n \cos^m x\ dx$ and substitute $u = \cos x$.

   (b) $\int (\cos^{2n+1}x) \sin^m x\ dx$ (odd power of cosine)

   Rewrite as $\int \cos x(1-\sin^2 x)^n \sin^m x\ dx$ and substitute $u = \sin x$.

   (c) $\int \sin^{2n}x \cos^{2m}x\ dx$ (even power of sine/cosine)

   Rewrite using half-angle identities as $\int [\frac{1}{2}(1-\cos(2x))]^n\ [\frac{1}{2}(1+\cos(2x)]^m\ dx$.

   (d) $\int \tan^{2n+1}x \sec^{m+1}x\ dx$ (odd power of tangent)

   Rewrite as $\int \tan^{2n}x \sec^m x(\tan x \sec x)\ dx$
   $= \int (\sec^2 x - 1)^n \sec^m x\ (\tan x \sec x)\ dx$, then substitute $u = \sec x$.

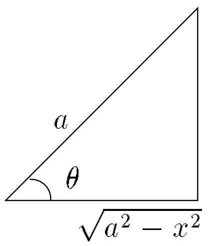
   (e) $\int \tan^m x \sec^{2(n+1)}x\ dx$ (even power of secant)

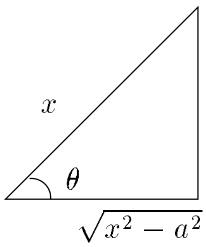
   Rewrite as $\int \sec^{2n} \tan^m x \sec^2 x\ dx$
   $= \int (1+\tan^2)^n \tan^m x\ (\sec^2 x)\ dx$, and substitute $u = \tan x$.

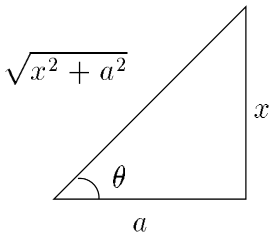
   (f) $\int \sec^{2n+1}x \tan^{2m}x\ dx$ (odd power of secant, even power of tangent)

   Rewrite as $\int \sec^{2n+1}x(\sec^2 - 1)^m\ dx$ and try INTEGRATION BY PARTS!

4. Trigonometric Substitution:







Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\ \sin\theta,\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\ \sec\theta,\ 0 \leq \theta \leq \frac{\pi}{2}$ or $\pi \leq \theta \leq \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\ \tan\theta,\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$

5. Partial Fractions

If the integral looks like:

$$\int \frac{p(x)}{q(x)}\ dx,$$

where $p(x), q(x)$ are both polynomials ...

(a) If degree($p(x)$) $\geq$ degree($q(x)$),simplify first with long division. Then with the remainder term, follow the advice below.

(b) Start by factoring $q(x)$ as much as possible.

(c) Rewrite $p(x)/q(x)$ as a sum of simpler functions:

For linear factors $(x-a)^n$ of $q(x)$, the sum will contain terms

$$\frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_n}{(x-a)^n}.$$

For irreducible quadratic factors $(x^2+ax+b)^n$ of $q(x)$, the sum will contain terms

$$\frac{A_1x + B_1}{(x^2+ax+b)} + \frac{A_2x + B_2}{(x^2+ax+b)^2} + \cdots + \frac{A_nx + B_n}{(x^2+ax+b)^n}.$$

(d) Now once you've simplified, integrate each term. The linear terms $(x-a)^n$ are easy to integrate with u-substitution. For irreducible quadratic terms (x^2+ax+b), you have to complete the square, then probably try to use the inverse trig. functions. For example, recall that

$$\frac{d}{dx}\left(\frac{1}{a}\right)tan^{-1}\left(\frac{x}{a}\right) = \frac{1}{a^2+x^2}.$$