SOLUTIONS TO 1

1. (26 points) Find the following definite or indefinite integrals:

a) $I = \int \cos x \ \sin^2 x \ dx$

Let $u = \sin(x)$. Then $du = \cos(x)\ dx$. So $I = \int u^2\ du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3(x) + C$.

b) $\int \frac{x\ dx}{(x-3)(x+5)}$

Use partial fractions to rewrite this integral as the sum of two integrals:

$$\frac{x}{(x-3)(x+5)} = \frac{A}{x-3} + \frac{B}{x+5}$$

So, $x = A(x+5) + B(x-3) = (A+B)x + (5A - 3B)$. Hence $A + B = 1$ and $5A - 3B = 0$. Next we solve for A and B: $B = 1 - A \Rightarrow 5A - 3(1-A) = 0 \Rightarrow 8A = 3 \Rightarrow A = \frac{3}{8} \Rightarrow B = \frac{5}{8}$. So,

$$\begin{aligned}\int \frac{x}{(x-3)(x+5)}\ dx &= \int \frac{3}{8(x-3)}\ dx + \int \frac{5}{8(x+5)}\ dx \\ &= \frac{3}{8}\ln|x-3| + \frac{5}{8}\ln|x+5| + C.\end{aligned}$$

c) $I = \int \cos\sqrt{x}\ dx$

Let $u = \sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}}\ dx$. So $dx = 2u\ du$ and $I = 2\int u\cos(u)\ du$.
Now let $w = u$ and $dv = \cos u\ du$. So $dw = du$ and $v = \sin u$.
Integrating by parts, we get $I = 2[u\sin u - \int \sin u\ du] = 2u\sin u - 2(-\cos u) + C = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$.

d) $I = \int_{-1}^{2} |xe^x|\ dx$.

The function xe^x is positive for $x > 0$ and negative for $x < 0$. So

$$|xe^x| = \begin{cases} -xe^x & \text{if} \quad x < 0 \\ \ \ xe^x & \text{if} \quad x \geq 0 \end{cases}$$

and the integral splits into two pieces: $I = \int_{-1}^{0} -xe^x\ dx + \int_{0}^{2} xe^x\ dx$.

To integrate $\int xe^x\ dx$ by parts, let $u = x$ and $dv = e^x\ dx$. Then $du = dx$ and $v = e^x$. So $\int xe^x\ dx = xe^x - \int e^x\ dx = xe^x - e^x + C$.
So by the evaluation theorem $I = -[xe^x - e^x]_{-1}^{0} + [xe^x - e^x]_{0}^{2} = -[(0e^0 - e^0) - (-1e^{-1} - e^{-1})] + [(2e^2 - e^2) - (0e^0 - e^0)]$. (If you choose to simplify this is $e^2 - \frac{2}{e} + 2$.)