

November 27, 2001

Instructions: 1) Write your name, your instructor's name, and your section number on the front cover of the exam booklet. Sign the academic integrity statement.

2) Show all your work on all questions and explain your answers carefully.

A correct answer without justification gets no credit.

3) Do not simplify your answers.

4) There are 7 problems worth a total of 100 points. (Problems 6 and 7 are on the back of this sheet.) Do them in whatever order you wish, but label each solution clearly.

5) No calculators are allowed on this exam.

Problems:

1. (20 points) Find the radius and interval of convergence of the following power series.

$$\text{a) } \sum_{n=0}^{\infty} \frac{x^{3n}}{n!} \qquad \text{b) } \sum_{n=1}^{\infty} \frac{(x-1)^n}{n}.$$

2. (20 points) a) Find a power series representation for the function

$$f(x) = \frac{x^2}{(1-x)^2}.$$

(Start by differentiating a suitable geometric series.)

b) Determine the radius of convergence of the series you found in part a).

c) Is the series you found in part a) the Maclaurin series of $f(x)$? Explain your answer.

3. (10 points) Using the definition of Taylor's series find the first three terms of the Taylor series for $\sin x$ centered at $a = \pi/4$. (The constant term counts as the first term. You may find it useful to remember that $\sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2}$.)

4. (8 points) Use Euler's method with step size $1/2$ to estimate $y(2)$, where $y(x)$ is the solution of the initial-value problem $y' = 3x - 2y + 1$, $y(1) = 2$. Remember to show your work.

5. (12 points) Solve the initial-value problem $\frac{dy}{dx} = 2x + 2xy^2$, $y(0) = 1$.

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6. (15 points) A tank contains 500 liters of pure water. Brine that contains 0.1 kilograms of salt per liter of water enters the tank at a rate of 5 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after t minutes?

7. (15 points) Match each differential equation with its direction field (labeled I–IV). Give reasons for your answers.

(a) $y' = x - y$, (b) $y' = 2x - 1$, (c) $y' = x^2 + y - 1$, (d) $y' = y$.

