Project Based Math 112, Fall 2001
Project #3 — The Sinister Serum of Dr. Jekyll
Due Last Class

Part 1

Mr. Edward Hyde
London, England
14 November, 1885

Math 112 Students
Cornell University
Ithaca, NY 14853

My Learned Associates,

You are all aware of my regrettable situation; that through a series of experiments, Dr. Henry Jekyll fragmented his personality, refining himself and creating me; that control has switched unpredictably between us and we have lived two separate lives in a shared body, to what extent that is possible. Knowing that our well-being is inexorably unified, we have formed an uneasy truce. That has ended. My alter ego has developed a stabilizing serum which would, when properly administered, prevent future personality changes. He means to suppress me forever!

I became aware that he was working towards such a serum from his research journal, which he leaves unhidden—perhaps he thinks me illiterate. He completed the serum today. I would have been lost but for a miraculous turn of fortune: as he was preparing to inject his first dose, control was exchanged. I have been granted an opportunity to save myself from oblivion, but my only hope is to master his invention. If I am to survive I must learn to use the stabilizing serum; I must seize control of this body. There is more at stake than my existence. The brilliant Dr. Jekyll is increasingly mad. I have considered taking my life to bring his demented and profane experiments to an end, but I have been too weak, too selfish. Soon, such drastic measures will either not be required, or not be possible.

The good doctor's notes should permit me to reproduce the serum without trouble, but the concentration of serum in my blood must remain within certain bounds for it to be both safe and effective. I am personally unable to determine an appropriate dose size and frequency. That is why I have turned to you. Your mathematical abilities far exceed my own; you are doubtlessly equal to the challenge. From Dr. Jekyll's journal I have surmised that the serum will be effective as long as the concentration is at least .5 mg/ml. However, above 1.5 mg/ml the serum may have adverse effects, and agove 2.5 mg/ml it would be fatal. The kidneys flush the serum from the blook at a rate proportional to its concentration. More specifically, if $b(t)$ is the concentration (in mg/ml) of serum in the blook after t hours, then $b'(t) = -.4b(t)$. Finally, the doctor and I have 5.1 Litres of blood running through our shared veins. I can only hope that no further information is needed.

I desperately await your response. I will pass the next several days in a room secured by locks with combinations known only to me; it is a crude obstruction and will buy me but a little time. This is my final battle with an adversary that I have come to respect and fear and who, until now, I believed that I could never escape. With your help, I may yet prevail.

Yours sincerely,

Mr. Edward Hyde

Part 2
One month later

Mr. Edward Hyde
London, England
14 December, 1885

Math 112 Students
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My Studious Benefactors,

I am unspeakably grateful for the help that you have given me. Your calculational efforts have allowed me to suppress the diabolical Dr. Jekyll for an entire month, but it has taken a toll. The frequent need to intravenously administer the stabilizing serum has left my arms covered in needle tracks that would impress a heroin addict and has prevented me from sleeping through the night—indeed, I live in constant fear of missing a nocturnal dose and disappearing forever into the abyss of Jekyll's demented mind.

A better solution may exist. I have carefully studied my alter ego's research journals; he was planning to produce a form of the serum to be taken orally. His notes are complete enough that I have managed to manufacture the potable elixir. Dr. Jekyll determined that the ingested serum would have an important advantage over the injected form; it is introduced into the bloodstream gradually, allowing larger doses to have prolonged effectiveness without causing an overdose. He tantalizingly scribbled in the margin of his notebook that the new serum needs only to be ingested three times per day, a vast improvement over my current situation.

Once again, all necessary information seems to be provided in the doctor's journal and, once again, my ignorance of mathematics prevents me from calculating an appropriate dosage. Therefore I must, once again, rely on your skills and charity.

This is what I know. If there are $s(t)$ grams of serum in the stomach at t hours, then $s'(t) = -.3\,s(t)$. Only 73% of the serum that is absorbed by the stomach enters the bloodstream; the rest is flushed from the system. Everything else is as before. In particular, the concentrations at which the serum is both safe and effective are unchanged, as is the rate at which the serum is removed from the blood. Is it true that the oral form of the serum requires consumption only once every eight hours, and if so, how much should I ingest?

My continued existence is in your capable hands.

Yours sincerely,

Mr. Edward Hyde

Questions

You should be sure to answer the following questions completely and correctly.

Part 1

(1) Let D be the size in grams of each dose and T be the time in hours between doses. Find values of D and T to ensure that the concentration of serum remains indefinitely between the *effective* and *nonlethal* limits (.5 mg/ml and 2.5 mg/ml). Make T as large as possible. Graph the concentration of serum in the blood over the course of several doses.

(2) Show that no values of D and T will indefinitely keep the concentration between the *effective* and *safe* limits (.5 mg/ml and 1.5 mg/ml).

(3) Intuitively, a large initial dose to establish a reasonable concentration of serum, followed by smaller doses to maintain the concentration, should give better results. Let I be the size in grams of the initial dose and D be the size in grams of subsequent doses. Find values of I, D and T to ensure that the concentration of serum remains indefinitely between the *effective* and *safe* limits. Again, make T as large as possible.

(4) Suggest a reasonable dosage schedule for the infamous and illustrious Mr. Hyde. Graph the concentration of serum in the blood over the course of several doses.

Part 2

(1) Write down and solve the differential equation for $b(t)$, the concentration in mg/ml of serum in the blood after t hours. The solution should depend on two initial conditions: the concentration of serum in the blood and the amount of serum in the stomach, both when $t = 0$.

(2) Find a dose D in grams which, when taken every eight hours, ensures that the concentration of serum in the blood remains indefinitely between the *effective* and *safe* limits. Remember that you have control over the initial conditions; you should figure out how to switch from your schedule for the injected serum to a schedule for the ingested serum. Graph the concentration of serum in the blood over the course of several doses.

(3) Show that there is no dose which, when taken every twelve hours, will ensure that the concentration of serum in the blood remains indefinitely between the *effective* and *safe* limits, even taking into account that you have control over the initial conditions. Hint: consider the long term concentration of serum in the blood.

Extra Credit

When did British scientists begin using the metric system? Can you find further anachronisms or other errors?