Abstract—Mechanical micro and nano scale devices are being increasingly used for realizing signal processing, logic or computing functions, which until recently were traditionally performed by electronics. Extremely low power consumption and robustness make mechanical micro structures attractive for implementation in autonomous distributed sensing systems. Heterodyning is one of the most important and indispensable signal processing techniques. In microelectromechanical heterodynes, the frequency mixing is commonly achieved using nonlinear forces which often limits scalability and the operational range of the device and may lead to undesired dynamic behavior. In this work we introduce an approach allowing purely mechanical, realization of the superheterodyne principle. The frequency mixing is based on the inertial coupling between two vibratory modes of the device. The architecture is distinguished by linearity of the mixing term with respect to the input inertial and the local oscillator signals. We demonstrate the mixing effect both theoretically and experimentally, using the devices fabricated from a silicon on insulator (SOI) wafer by deep reactive ion etching (DRIE). We show the applicability of the device as a mechanical low frequency vibration sensor. The substrate vibration frequencies down to 25 Hz were measured using a device with a fundamental mode frequency of 4700 Hz.

Index Terms—Heterodyne, vibrations sensing, variable inertia, inertial coupling, mechanical signal processing.

INTRODUCTION

HETERODYNING is one of the key elements in modern signal processing [1]. This technique permits mixing of two signals with different frequencies $\omega_1$ and $\omega_2$ to produce two new signals with the frequencies $\omega_1 - \omega_2$ and $\omega_1 + \omega_2$. This principle is implemented in countless applications, for example, radio and optical [2] communication, frequency measurements [3] and high precision optical detection [4]. Heterodyning is routinely realized using electronic elements operated in a nonlinear regime [5].

Advances in micro- and nanotechnology opened new opportunities in design and operation of extremely downscaled machines with previously unthinkable performance. This stimulated a renewed interest to (electro-)mechanical realization of various signal processing functions, which in the past were reserved solely for electronics. Micro- and nanoelectromechanical systems (MEMS/NEMS) were successfully implemented not only as analog radio frequency (RF) components such as filters, [6] multiplexers, [7] counters [8] and frequency control and timing devices, [9] but also as micro mechanical memories [10] and logic elements [11], [12]. While their penetration into the electronic components market is still low, mechanical devices offer unique functional advantages making them attractive. Resonant micro- and nanomechanical structures often exhibit smaller losses, lower power consumption, wider operational range and better frequency stability than their electronic counterparts, while preserving comparable size and operational frequencies [9], [13].

Emerging of new technologies, such as Internet of Things (IoT) or wearables for personalized health care, both essentially based on distributed data collecting networks, imposes new demanding power consumption, integrability and overall size requirements to the sensing systems. The use of mechanical structures serving simultaneously as a sensing element and as a logic data processing device may reduce or even eliminate the need in electronic data processing, therefore reducing dissipated power to minimum. As an example, one can mention threshold inertial switches [14] or contact-based vibration detectors for wireless wake-up monitoring sensors, [15] both serving as mechanical alternatives to the computationally intensive post-processing of the time history data collected by accelerometers. In addition, mechanical structures are generally less prone to the electromagnetic interference (EMI) and can be operated in harsh environments such as elevated temperatures or radioactivity [16]. Within the long standing arena of analog computing, [17], [18] mechanical devices are considered as one of the possible approaches, which may pave the way to zero power reversible computing [19].

Similarly to mechanical computing, the idea of mechanical heterodyne has a long history. Complex macro-scale mechanisms realizing the heterodyne principle were already reported back in the sixties [20]. At the micro- and nano-scale, electromechanical MEMS/NEMS heterodynes reported so far exploited nonlinear electrostatic[21]–[27], electro thermal [28] or optical [29] forces to serve as a frequency multiplying element. For example, the intrinsic structure of the electrostatic forces $F_e = V^2(t)G(u)$, where $V(t)$ is a time-dependent voltage and $G(u)$ is a nonlinear function of the device’s response $u(t)$, allows mixing by means of multi-frequency excitation, when $V = V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t)$ [21]–[25]. A superheterodyne device, which exploits an electrostatic
coupling between the different parts of the multi degrees of freedom (DOF) structure, each vibrating at different frequencies, was reported in [30], [31]. Linearity of the frequency multiplying term was achieved by implementing a comb drive transducer with linearly varying electrodes length. In all of these cases, the heterodyning effect is based on the nature of the actuation forces, rather than on mechanical effects.

Here we report on an approach allowing purely mechanical realization of the superheterodyne principle. The device is based on the inertial geometric coupling (it is to say, the coupling appearing due to the changes in the device geometry influencing the inertial terms) between the translational and tilting vibrational modes of a proof mass attached to an oscillating substrate. In our architecture, the translational vibration creates a time-harmonic offset between the centroid of the proof mass and the tilting axis. As a result, the actuating moment driving the tilting vibrations and serving as the frequency mixing element emerges as a product between the inertial force associated with the substrate acceleration and the time-harmonic translational displacement. In other words, the translational motion serves as a local oscillator providing a carrier signal, whereas the tilting mode vibrations are excited at a fixed intermediate frequency.

We demonstrate, both theoretically and experimentally, the feasibility of the suggested concept. We also show that our device can be used as a sensor for low frequency vibration detection. In many applications, such as structural health monitoring [32], it is required to detect mechanical vibratory signals at low, down to tens or even few Hz, frequencies. In these applications, spectral signature of a large-scale host structures, such as bridges, [33], wind turbines [34], or airplane wings [35], is monitored in order to detect possible accumulated damage. The use of accelerometers [33] or fiber Bragg grating (FBG) optical sensors [35] is not always convenient since it requires computationally intensive, power-consuming, Fourier processing of the acquired time-history data. On the other hand, the direct frequency detection through resonant operation of micro mechanical filters is challenging due to large mismatch between the acceptable (from the strength point of view) micro devices natural frequencies and the values to be measured. In this context, the ability of the suggested heterodyne device to perform upconversion of the input signal frequency enables detection of the macro-scale structures vibration using miniature low power consumption and low cost MEMS devices, therefore overcoming the inherent frequency mismatch issue.

MODEL

In order to highlight the operational principle of the mechanical superheterodyne and to provide an insight into its dynamic behavior, we first consider the simplest two DOF, mass-spring model of the device, Fig. 1(a),(b). A rigid frame with the central moment of inertia $I_F$ is attached to the vibrating substrate by a massless linearly elastic torsional spring with the stiffness $k_{\theta}$. The frame performs a tilting motion with the angle $\theta(t)$, where $t$ is time. (Hereafter, hats ( ) denote dimensional quantities.) A rigid proof mass $M$ with the central moment

![Fig. 1. Lumped two DOF model of the device in the initial unreformed (a) and the deformed (b) configurations. $M\dot{a}(t)$ is the resultant of the inertial force acting at the proof mass centroid $C$. (c) Block diagram of the superheterodyne involving three participating signals (insets): the high frequency actuating force carrier signal $F(t)$ (drive mode), the low frequency input acceleration signal (sense mode) $a(t)$ and the modulated output tilting signal $\dot{\theta}(t)$. (d) Normalized analytic Fourier Transform of linearized model solution, eq. (9), for linear actuation frequency of $\omega = 1$ Fourier Transform of the linearized model solution, eq. (9), for the normalized (by the tilting mode natural frequency) actuation frequency of $\omega = 1$ and two different non-dimensional substrate acceleration frequencies: $\Omega = \Omega_1 = 0.01$ and $\Omega = \Omega_2 = 0.02$.]

of inertia $I_M$ is attached to the frame by a linearly elastic spring with the stiffness $k_u$. The mass performs a translational motion $\dot{u}(t)$ within the plane of the frame and the tilting motion together with the frame. For the sake of convenience, the mass and the frame are shown in Fig. 1(a),(b) with a certain vertical offset between them. In the actual device the centroids of the mass and of the frame are located within the same $(xy)$ plane. At this stage, we assume that the geometry is perfectly symmetric and that in the initial undeformed state, the mass centroid $C$ is located at the tilting axis $x$. The translational vibrations of the mass are excited by an actuating force $\dot{F}(t)$, which is directed within the plane of the frame. Due to the substrate vibration with the acceleration $\ddot{a}(t)$, the proof mass is also subject to an inertial force with the resultant $M\dot{a}$ acting at the mass centroid $C$, Fig. 1(b).

The equations of motion are developed using a variational principle (see Supplementary information for details). By
substituting the kinetic and the potential energies of the system
\[ T = \frac{1}{2} (I_F + I_M + M \dot{u}^2) \left( \frac{d\theta}{dt} \right)^2 + \frac{1}{2} M \left( \frac{d\hat{u}}{dt} \right)^2 \]

\[ U = \frac{1}{2} k_\theta \theta^2 + \frac{1}{2} k_u \dot{u}^2 + M \dot{u} \sin(\theta) - F \hat{u} \]

along with the Rayleigh dissipation function \( F = c_0 \left( \frac{d\theta}{dt} \right)^2 / 2 + c_u \left( \frac{d\hat{u}}{dt} \right)^2 / 2 \), (where \( c_0 \) and \( c_u \) are the viscous damping coefficients associated with the tilting and the translational motions, respectively) into the Lagrange’s equation of the second kind, and assuming \( \sin(\theta) \approx \theta \), \( \cos(\theta) \approx 1 \), we obtain two equations governing the system motion
\[ M \frac{d^2\hat{u}}{dt^2} + c_u \frac{d\hat{u}}{dt} + \left( k_u - M \frac{d\theta^2}{dt^2} \right) \hat{u} + M \ddot{\theta} = \hat{F} \]  

(2)

\[ (I_F + I_M) \frac{d^2\theta}{dt^2} + M \frac{d}{dt} \left( \frac{d\theta}{dt} \right) + c_0 \frac{d\theta}{dt} + k_\theta \theta + M \ddot{u} = 0 \]  

(3)

For convenience, we re-write Eqs. (2) and (3) in the non-dimensional form
\[ \ddot{u} + \frac{\omega_u}{Q_u} \dot{u} + (\omega_u^2 - \hat{\theta}^2) u + a \theta = F \omega_u^2 \]  

(4)

\[ \frac{d}{dt} \left( (1 + u^2) \hat{\theta} \right) + \frac{1}{Q_\theta} \hat{\theta} + \theta + auu = 0 \]  

(5)

where
\[ u = \frac{\ddot{u}}{\hat{u}}, \quad t = \frac{\hat{t}}{\tau}, \quad \omega_u = \frac{\hat{\omega}_u}{\hat{\omega}_r}, \quad Q_u = \sqrt{k_u M}, \quad Q_\theta = \sqrt{k_\theta (I_F + I_M)} \]

(6)

\[ a = \frac{\hat{a} \tau}{\hat{\omega}_r}, \quad F = \frac{\hat{F}}{k_u} \]

Here \( \omega_u = \sqrt{k_u/M} \) and \( \hat{\omega}_u = \sqrt{k_\theta / (I_F + I_M)} \) are, respectively, the translational and the tilting mode natural frequencies corresponding to the linear uncoupled unforced counterparts of Eqs. (4), and (5) (i.e., to the case of \( \hat{\theta} = 0 \), \( F = 0 \) in Eq. (4), \( u^2 = 0 \) in Eq. (5) and \( a = 0 \) in both equations); \( Q_u, Q_\theta \) are the corresponding quality factors. In accordance with Eq. (6), the gyration radius \( r = \sqrt{(I_F + I_M)/M} \) and the period of the free tilting vibrations \( \tau = 1/\hat{\omega}_u \) are used as the natural units of the displacement and time, respectively. Hereafter, the overdot denotes derivatives with respect to the non-dimensional time \( \cdot = d/dt \).

The system Eqs. (4), (5) incorporates both linear and nonlinear coupling between the DOFs. The linear coupling terms are parameterized by the time-dependent substrate acceleration \( a(t) \). The nonlinear coupling manifests itself in the centrifugal softening term \( \hat{\theta}^2 \) in Eq. (4) and in the variable moment of inertia \( 1 + u^2 \) in Eq. (5). Presence of nonlinearities may lead to rich dynamic behavior of the device, while time-dependent coefficients may result in interesting parametric resonant responses [36], [37].

Linearity of the mixing terms \( au, a \theta \) in the carrier, input and the output signals amplitudes offers a possibility to study the heterodyne effect by considering only the linear counterpart of Eqs. (4) and (5). We simplify our model by neglecting \( \hat{\theta}^2 \) (since \( \theta \ll 1 \) and \( \omega_1 = 1 \)) and \( a \theta \) in Eq. (4) (since it is small compared to \( F \omega_u^2 \)). In Eq. (5), we assume time-invariant inertia and neglect the \( u^2 \ll 1 \) term. However, we preserve the inertial coupling in Eq. (5), which is the only driving term exciting the tilting motion.

In the harmonic excitation case \( F = F_0 \cos(\omega t) \) simplified Eqs. (4), (5) are
\[ \ddot{u} + \frac{\omega_u}{Q_u} \dot{u} + \omega_u^2 u = F_0 \omega_u^2 \cos(\omega t) \]

(7)

\[ \hat{\theta} + \frac{1}{Q_\theta} \hat{\theta} + \theta = -u_a \cos(\Omega t) \]

(8)

By looking at the Eqs. (7), (8) we may immediately conclude that the mixing takes place and the tilting motion is excited at the frequencies \( \omega + \Omega \) and \( \omega - \Omega \), as depicted in the schematic block diagram, Fig. 1(c). The effect here is a true heterodyne (rather than beats [3]), it is purely geometric and is not influenced by the nature of the actuating force \( F \). By substituting the solution \( u(t) = u_0 \cos(\omega t - \phi_u) \) of Eq. (7) into Eq. (8) and by solving the resulting equation in terms of \( \theta \) we obtain the output signal
\[ \theta(t) = u_0 a_0 \left[ G_1 \cos \left( \left( \omega - \Omega \right) t - \phi_u - \phi_1 - \frac{\pi}{2} \right) \right. \]

\[ + G_2 \cos \left( \left( \omega + \Omega \right) t - \phi_u - \phi_2 - \frac{\pi}{2} \right) \]

(9)

where
\[ u_0 = F_0 G_u \]

\[ G_u(\omega) = \left[ \left( 1 - \omega_u^2 \right) + \frac{\omega_u^2 Q_u^2 \omega_u^2}{2} \right]^{-1/2} \]

(10)

\[ \tan(\phi_u) = \frac{\omega_u}{Q_u \omega_u} \left[ 1 - \omega_u^2 \right]^{-1} \]

\[ G_I(\omega) = \left[ \left( 1 - \omega_i^2 \right) + \frac{\omega_i^2 Q_i^2}{2} \right]^{-1/2} \]

(11)

\[ \tan(\phi_i) = \frac{\omega_i}{Q_i} \left[ 1 - \omega_i^2 \right]^{-1} \]

and \( \omega_i = \omega + (-1)^i \Omega, \quad (i = 1, 2) \). In accordance with Eqs. (9)-(11), the resonant tilting response occurs at two modulated frequencies \( (1 \pm \Omega) \sqrt{1 - 1/(2Q_u^2)} \), therefore demonstrating mixing. Moreover, the magnitude of the tilting mode vibrations is linear in terms of the acceleration \( a_0 \) and the translational \( u_0 \) amplitudes.

Figure 1(d) shows the analytic fast Fourier transform (FFT) of the solution Eqs. (9)-(11). The results correspond to the case of the high frequency translational mode excitation at \( \omega_u = 0.85 \) and the low frequency inertial excitation at the substrate non-dimensional frequencies \( \Omega = 0.01 \) and \( \Omega = 0.02 \). Two peaks correspond to the non-dimensional driving frequency of \( \omega = 1 \pm \Omega \) illustrating the mixing effect. To assure that the solution of the simplified Eqs. (7), (8) are consistent with the results obtained using Eqs. (4), (5), we solved numerically Eqs. (7), (8) and found that for the considered range of parameters, the analytical solution (9) is in an excellent agreement with the numerical data. It
is to say, the influence of the parametric and nonlinear terms in Eqs. (7), (8) on the mixing effect is minor.

Due to its simplicity and availability of the closed form analytical solution, the lumped model, Eqs. (7), (8) is convenient for the heterodyne effect analysis in the device under consideration. It can be used also at the initial stages of the device development for evaluation of the design parameters. However, our experimental results show additional, not predicted by Eqs. (9)-(11), resonant peaks at certain modulated frequencies.

We attribute the presence of these peaks to the influence of the geometric imperfections, which are unavoidable due to the low tolerances of micro machining, and may result in additional dynamic couplings and comb drive related forcing terms.

In order to investigate the influence of the imperfections, we expand the model to include an initial vertical (z-direction) offset $\delta$ between the centroid of the mass and the $xy$ plane, (see Fig S.1 in the Supplementary materials). Moreover, in the present work we use an electrostatic force generated by two comb drive transducers to excite the translational motion of the mass. Since the electrostatic force may affect the device’s response, our modified model accounts also for the influence of the fringing electrostatic fields, which results in appearance of the time-dependent parameterized by the voltage (effective) stiffness coefficients in the governing equations. Note that in the actual device, many other sources of imperfections, such as asymmetry in the mass geometry (resulting in an additional initial in-plane offset) or in the transducers voltages may exist. Since the influence of these imperfections is, at least qualitatively, similar to that of $\delta$, we limit our consideration to the case when only the vertical offset is present. After taking into account the influence of imperfections, the following system of equations was obtained (the detailed derivation is found in the Supplementary materials)

$$
\ddot{u} + \frac{\omega_n}{Q_u} \dot{u} + (\omega_n^2 - \theta^2)u + \left[ a - \beta \eta \left( y_e + L_e \right) (V_R^2 + V_L^2) \right] \theta + \delta \ddot{\theta} = \beta h (V_R^2 - V_L^2)
$$

$$(1 + \nu^2) \ddot{\theta} + 2u \dot{\theta} \dot{u} + \frac{1}{Q_\theta} \dot{\theta} + \left[ 1 + \beta \nu L_e y_e^2 (V_R^2 + V_L^2) \right] \theta + \left[ a - \beta \eta \left( y_e + L_e \right) (V_R^2 + V_L^2) \right] u + \delta \ddot{u} = \beta \eta y_e L_e (V_R^2 - V_L^2)$$

Here $\delta = \dot{\delta}/r$ is the normalized vertical offset, $\eta$ and $\nu$ are the non-dimensional parameters associated with the fringing electrostatic fields, $L_e$ is the initial overlap between the comb drive electrodes and $y_e$ is the distance between the midpoint of the electrodes overlap area and the torsion axis $x$, Fig. 2(a). In addition, $\beta = N \varepsilon_0 V_R^2 \omega_n^2 / (g_0 k_n)$ is the voltage parameter and $V_R$ and $V_L$ are the voltages applied to the right and to the left transducer, respectively. Equations (12), (13) suggest that since $V_R^2 - V_L^2 = 2V_{DC} V_{AC} \cos(2\omega t)$, the resonant excitation of the device takes place when the excitation frequency $\omega$ is close to the effective (influenced by the voltage and by the centrifugal terms) resonant frequencies associated with the translational or the tilting modes. The presence of the $V_R^2 + V_L^2 = 2V_{DC} + V_{AC}(1 + \cos(2\omega t))$ terms may result in parametric excitation of the system. Nonlinear quadratic in $u$ and $\theta$ terms may lead to the emergence of additional sidebands peaks in the response spectrum. We emphasize that in accordance with Eq. (13) in our device, in contrast to [31], even in the presence of the actuating force, the desired mixing effect is due to the inertial rather than the electrostatic coupling.

**Device Architecture**

The device shown in Fig. 2(a) contains a frame performing a tilting motion around the $x$ axis and two identical proof masses designed to move in a translational, in-phase, mode within the plane of the frame (in the $y$ direction) and therefore also undergoing tilting together with the frame. The frame is attached to the unmovable anchors by two double torsion bars. The design based on the double bars allows to increase the in-plane rotational stiffness of the frame suspension. Each of the proof masses is elastically connected to the frame by two folded beam suspensions. The thickness of the device in the out-of-plane $z$ direction is much higher than the width of the folded beams and of the torsion bars. This assures high out-of-plane stiffness of the device. To excite the translational vibrations of the proof mass, we use two comb drive transducers positioned at the outer edges of each of the masses. The third transducer is located at the center of the device, between the two masses. It was not used for actuation in our experiments. The substrate that the entire frame-masses assembly is attached to is forced to vibrate in the out-of-plane $z$ direction.

The devices were fabricated at Rafael Micro Systems center (RAMS) using an in-house wafer level deep reactive ion etching (DRIE) based process (SPTS Pegasus tool) and released by hydrofluoric acid (HF) vapor etching (Primaxx VHF tool). As a starting material, 150 mm highly-doped (P-type Boron, resistivity of 0.01-0.02 $\Omega$cm) SOI wafers with (100) upper surface orientation, 70 $\mu$m thick conductive device layer, 2 $\mu$m thick buried oxide (BOX) and 400 $\mu$m thick conductive handle were used. Metal contact pads were defined by evaporating 0.7 $\mu$m thick layer of aluminum. The torsion bars and the folded beams are oriented in the crystallographic $<110>$ direction of Si. The nominal, “as designed”, length and width of the torsion bars is $L_t = 340$ $\mu$m and $b_t = 25$ $\mu$m, respectively, the distance between the centerlines of the double torsion bars is $e = 220$ $\mu$m. The length and the width of the folded suspension beams are $L_f = 765$ $\mu$m and $b_f = 20$ $\mu$m, respectively. Each comb drive actuator contained 149 electrodes, the distance between the electrodes is $g_0 = 5$ $\mu$m. The fabricated device, which was used in the experiments, is shown in Fig. 2(b).

**Model Results**

First, a three-dimensional finite element (FE) model of the device was built (ANSYS package) and four lowest natural frequencies were obtained using the eigenvalue solver (see 48 for details). The model accounts for anisotropy of Si and
incorporates actual, measured, geometric parameters of the device, which differ from the nominal values due to fabrication tolerances. The calculated natural frequencies (convergent up to the sub-percent relative error) corresponding to the in-phase translational and tilting modes were \(f_u = 4450 \text{ Hz}\) and \(f_\theta = 4920 \text{ Hz}\), respectively. Frequencies corresponding to the undesired anti-phase translational and out-of plane modes were 4594 Hz and 6453 Hz, respectively. The smallest split of 144 Hz was obtained between the in-phase and anti-phase translational modes frequencies. In our experiments, the devices demonstrated quality factors of the order of \(Q \approx 2000\). For this reason, the separations between the natural frequencies were sufficient to assure that only the desired vibrational modes are excited. The stiffnesses associated with the in-phase translational and the tilting modes were also calculated and were found to be \(k_u = 344 \text{ N/m}\) and \(k_\theta = 63.4 \times 10^{-6} \text{ Nm}\), respectively. The mass and the combined moment of inertia around the \(x\) axis (in the initial undeformed configuration) of the two proof masses system were estimated to be \(M_m = 0.440 \times 10^{-6} \text{ kg}\) and \(I_m = 53.110 \times 10^{-15} \text{ kgm}^2\), respectively. The frame moment of inertia around the tilting axis was \(I_F = 13.240 \times 10^{-15} \text{ kgm}^2\).

To study the influence of the nonlinear coupling Eqs. (4), (5) completed by zero initial conditions were solved numerically by using the Runge-Kutta-Fehlberg ordinary differential equations solver implemented in Maple. The amplitude of the steady state response was obtained for differing values of the driving frequency \(\omega\). Our calculations show that for the adopted realistic device parameters, the results are practically identical to those given by the simplified linear model, Eqs. (7), (8). The calculated peak amplitudes and frequencies differs from the analytically obtained values, Eqs. (9)-(11), by less than 0.001% and 0.01%, respectively.

The influence of imperfections in the device geometry, along with the role of the fringing fields electrostatic force, were explored by numerically solving Eqs. (12), (13). Computation time was chosen to be sufficiently long to eliminate the contribution of the initial conditions and of the transient effects to the high-Q device response. In addition, the FFT of the calculated signal was carried out using only the second part of the time series not affected by the initial conditions. Consistently with our experimental protocols, numerical experiments of two types were carried out. In the first scenario, the inertial carrier signal associated with the substrate acceleration was applied at \(\Omega = 1\), which is the tilting mode natural frequency. The translational motion was actuated in a quasi static regime, at \(\omega = 0.02 \ll \omega_u\). The result is depicted in Fig. 3(a) where the fast Fourier transform (FFT) of the numerically obtained tilting angle time series is shown. The presence of additional peaks at \(\Omega = 1 \pm 3\omega\) is attributed to the influence of nonlinear terms in the system of coupled Eqs. (12), (13). In the second operational scenario, the electrostatic force was applied at \(\omega = 0.85\) and served as the carrier signal, while the inertial force was applied at the low frequency of \(\Omega = 0.02\). The FFT plot of the tilting angle time series, Fig. 3(b), reveals two peaks at the modulated frequencies \(\omega_u \pm \Omega\). The peak at \(\omega = 0.85\) is due to the direct forcing term in the right hand side of Eqs. (12), (13).

**Experiment**

**Experimental setup**

In order to apply an inertial signal, the device was attached to an external piezoelectric actuator Piezostacks PST1000/16/20VS25 (Piezomechanik GmbH), providing an oscillatory acceleration in the out of plane direction. The actuator was connected to the voltage source through a voltage amplifier (F10A, FLC Electronics). Before running the experiments, we characterized the acceleration signal provided by piezoelectric actuator and found it to be harmonic. The harmonic voltage signal was applied to the left and the right comb drive transducers in such a way that \(V_L = V_{DC} + V_{AC}\cos(\omega t)\) and \(V_R = V_{DC} - V_{AC}\cos(\omega t)\). The device was placed inside the LCC ceramic package, where the signals were passed to the device through a compatible socket and wirebonding. The piezo actuator with the attached device was mounted on the wafer prober (Karl Suss PM5) equipped with the Mitutoyo FS70 microscope with a 20 ×/WD 20 mm long working distance lens. The prober was mounted on
Fig. 3. Model results considering imperfection effects, by meaning of FFT of $\theta$ response for (a) inertial input as a carrier ($\Omega \approx 1$) modulated by the electrostatic driving force ($\omega \approx 0.02$), and (b) driving actuation $\omega$ as a carrier ($\omega \approx 0.85$) with modulation by low frequency inertial signal ($\Omega \approx 0.02$). Insets show time history for both cases.

The goal of the experiments was two fold. The first was to demonstrate the heterodyne effect in the device under consideration and to explore its main features. The second goal was to show that the proposed device can serve as a vibration sensor able to detect frequencies much lower than the device’s own natural frequencies. Before carrying out the mixing experiments, the natural frequencies and corresponding quality factors associated with the in-phase translational and the tilting motions were measured. The tilting mode frequency was obtained by applying a transient inertial signal to the device, followed by the FFT of the recorded time series. The in-plane frequency was registered by applying a harmonic voltage to the comb transducer at a wide range of frequencies and using FFT of the measured response to identify the resonant peak. The in-phase translational and tilting frequencies were measured to be $\approx 4078$ Hz and $\approx 4700$ Hz, respectively, and are lower than the corresponding calculated values of $4450$ Hz and $4920$ Hz. The discrepancy is attributed to the uncertainty in the device geometry, especially in the parameters of the narrow, $\approx 20$ $\mu$m wide, folded suspension beams and $\approx 25$ $\mu$m wide torsional bars defining the device stiffness. Due to their small dimensions, these elements are prone to a significant influence of the fabrication-related deviations from the nominal geometry, such as an over etch or side wall angle resulting from the DRIE process. The quality factor associated with the small amplitude tilting motion was measured to be $Q_\theta \approx 2000$, were damping is caused mainly by air damping for small amplitude in free path, and anchor losses.

During our experiments, the devices were operated using two scenarios. In the first case, the acceleration signal was used as a carrier, while the electrostatic force was applied at a low frequency, well below the tilting resonance. The harmonic voltage signal with the amplitude of 50 V and at the frequency of $\approx 4700$ Hz was supplied to the shaker. The unipolar harmonic voltage signal with the offset of $V_{DC}=15$ V, amplitude of $V_{AC}=15$ V and at the frequencies of 100 Hz, 125 Hz, 150 Hz and 200 Hz was supplied to the comb drive transducers. The frequency responses, in terms of the out-of-plane velocity, are shown in Fig. 4. The presence of the side peaks corresponding to the modulated signals is clearly observed. The presence of the additional side bands in Fig. 4 is attributed to the influence of the nonlinear electrostatic and inertial (centrifugal) coupling between the tilting and the translational motions, nonlinear variable inertia and nonlinearity of the driving electrostatic force. Specifically, since the stationary electrodes of the comb drive transducer are attached to the substrate, the tilting and the translational motions are coupled through the electrostatic force (see Eqs. (S.18), (S.19) in the supplementary materials). This coupling, along with imperfections and the nonlinear variable inertia, Eqs. (12), (13), may result a possibly not purely harmonic frequency translational motion signal $u(t)$. Multiplication of the high frequency harmonic inertial signal $u(t)$ with the non-harmonic $u(t)$ can be seen in Fig. 4. In accordance with Figs. 4 the frequency separations between the adjacent major sidebands are the multipliers of the low frequency signal, implying that the heterodyning affects not only in the lowest harmonics but also in the sidebands. By subdividing the measured out-of-plane velocities by $\omega$, the peak values amplitudes of the out-of-plane vibrations were estimated to be $\approx 0.34$ $\mu$m for the central resonant peak...
frequency of $\approx 4700$ Hz and $\approx 3.5$ nm at the frequencies of the side bands. By further subdividing these values by the distance of $\approx 1400$ $\mu$m between the measurement point (the red dot in Fig. 2(b)) and the tilting axis, the corresponding angles were $\approx 0.24$ mrad and $\approx 2.5$ $\mu$rad.

In the second vibration sensing experiment, a low frequency inertial force was imposed on the device by the shaker, while the high frequency carrier was provided by the comb drive transducers. The amplitude of the voltage signal supplied to the shaker was 50 V, the frequency varied between 25 Hz and up to 125 Hz with the 25 Hz increments. By measuring the substrate out-of-plane velocities and subdividing the measured values by $\Omega$, the resulting accelerations were estimated to be in the range between 1 mg and up to 37 mg. The voltage signal feed into the comb drive transducers was at the resonant frequency $\approx 4078$ of the in-phase translational mode. The offset and the amplitude of the signal were $V_{DC} = V_{AC} = 15$ V. The device spectrum shown in Fig. 5 indicates that the frequency intervals separating the sidebands and the carrier peak (corresponding to the tilting mode resonant frequency) are equal to the substrate vibration frequency. In contrast to the case shown in Fig. (4), the low frequency inertial (acceleration) signal is harmonic, the influence of the high-frequency components of the non-harmonic translational response on the mixing is not pronounced and no additional sidebands are observed in Fig. 5. These results, apart from demonstrating the heterodyne effect, suggest that the device can serve as an efficient sensor for detecting a very low frequency of mechanical vibrations. Note that the superheterodyne based detection of the frequencies with the kHz range was reported in [31], [38]. Here we demonstrate the detection of the much lower frequency of $\approx 25$ Hz, which is 0.053% of the tilting mode harmonic.

**DISCUSSION**

The heterodyne architecture introduced in the present work relies on the inertial coupling between the two vibratory modes of the structure, rather than electrostatic or other nonlinear coupling. In our device, the measured quantity serving as an output is the tilting angle of the proof mass, while the translational motion provides a time-harmonic offset between the tilting axis and the mass centroid, where the inertial force resultant is applied. The frequency multiplying term appears as a product between the inertial force engendered by the vibrating substrate acceleration and the translational displacement.

**Fig. 4.** Experimental tilting response spectrum. The out-of-plane velocity associated with the tilting motion is shown for the constant shaker frequency of $\approx 4700$ Hz and different comb drive signal frequencies: (a) $\approx 25$ Hz; (b) $\approx 75$ Hz; (c) $\approx 100$ Hz and (d) $\approx 125$ Hz.

**Fig. 5.** Experimental tilting response spectrum. The out-of-plane velocity associated with the tilting motion is shown for the constant electrostatic force frequency of $\approx 4078$ Hz and different shaker frequencies: (a) $\approx 25$ Hz; (b) $\approx 75$ Hz; (c) $\approx 100$ Hz and (d) $\approx 125$ Hz (d). Inset in (c) shows the peak in the spectrum corresponding to the in-phase translational mode natural frequency.
As a result, the mixing term, which is the only term driving the tilting vibrations, is linear in terms of the substrate acceleration and the proof mass displacement. While the influence of nonlinearities on the device response can in principle become more pronounced at higher vibrational amplitudes, the linearity of the mixing term with respect to the substrate acceleration (the input) and the translational displacement is still preserved. The mechanical, geometric, nature of the mixing introduced in this work has several beneficial consequences. In contrast to the electrostatic mixing, our device is not prone to the so called pull-in instability, which often limits the stable amplitude range of the devices. Moreover, since the mixing is mechanical, it is not affected by the nature of the actuating forces driving the proof mass translational vibrations. Despite the fact that electrostatic actuation by a comb drive transducer was implemented in the present work, any other driving mechanism such as magnetic, piezoelectric, or inertial, can be used for this purpose. This expands the design space and offers more freedom in the choice of materials and processing approaches, which can be used for the device fabrication. Apart from the actuation, the mixing itself does not require any electric input (such as voltage or current), which may further reduce power consumption and make the device to be attractive for autonomous applications. More importantly, the mechanical nature of the mixing makes it invariant to the scale - the device can be up- or downscaled without affecting its functionality. This is in contrast to the heterodyne devices based on the nonlinear electrostatic forces, which generally cannot be scaled up. Note in passing that the same mechanical, based on the inertial coupling, heterodyne concept can be realized in the framework of other architectures. As an example, one can mention easily a down scalable micro or nano cantilever device with a mass elastically attached to the beam and moving along it. From the modeling point of view, the linearity of the mixing term allows to use for the heterodyne effect analysis, the simplest linear two DOF model, which allows a closed form solution and, despite its simplicity, still predicts heterodyning. The model can be conveniently used for the design purposes, as well as for the analysis of the influence of various design and operation parameters on mixing.

One of the distinguishing features of micromechanical devices is their unique ability to combine sensing and signal processing or logic functions within the same device. This allows significant reduction, or even elimination of a costly and time consuming post processing, and is especially attractive in the scenarios when the sensed signal is of a mechanical character. In this context, the inertial character of the mixing mechanism reported in this work opens interesting possibilities for implementation of the device in various applications. One of the possibilities explored in the present work is the low frequency vibration sensing, which, for example, can be incorporated into structural health monitoring systems. Direct MEMS-based mechanical resonant vibration sensing (mechanical Fourier transform) at the frequencies of the order of several tens or even hundreds of Hz is often challenging, since micro machined devices with such a low natural frequency are too fragile to be practical and are prone to handling-related damages. The use of accelerometers operated in a quasi-static mode requires complex processing of the time-series data and is not always suitable. Here we show both experimentally and by using the model, that the device can be applied for ultra low frequency vibration sensing. Substrate vibrations at the frequency as low as \( \approx 25 \) Hz were measured using a robust and manufacturable device with the natural frequency as high as \( \approx 4700 \) Hz. The minimal detectable frequency and the frequency resolution can be improved by operating the device at higher quality factors using appropriate packaging. We emphasize that in our experiments the device was actually operated as a superheterodyne when the input signal (substrate vibrations) frequencies were up-converted to the fixed intermediate frequency, namely the natural frequency associated with the tilting mode. The proof mass vibrating in the translational mode served as a local oscillator. It should be mentioned that while our calculations and experiments were carried out for the signals close to one of the natural frequencies of the system the heterodyne effect is possible at any combination of the inertial and the driving signal frequencies. However, for the energetic efficiency considerations, it is preferable to exploit the resonant amplification of the device response. This is especially suitable for micro- and nanomechanical resonant devices, which could demonstrate extremely high quality factors [6-9].

The mechanical mixing paradigm introduced here can be also exploited in other sensors. For example, the same measurement concept, which was used for the frequency measurement of the vibrating substrate, can be directly used for the spectral analysis of other forces, such as acoustic or electromagnetic, or even van der Walls forces in atomic force microscope (AFM). The device, with certain design modifications, can be used as a pressure sensor, microphone, electric/magnetic field sensor or even for speech recognition [39]. In all these implementations, the product of a time-harmonic force (of any kind) and of the proof mass displacement will result in a heterodyne effect and in the resonant excitation of the tilting vibrations. The device can also be used as a resonant accelerometer operated in the frequency monitoring scenario. For example, in the case of \( a = \text{const} \), the eigenvalues of the linear undamped and unforced counterpart of Eqs. (4) and (5)

\[
\lambda_{1,2} = (\omega_a^2 + 1 \pm \sqrt{(\omega_a^2 - 1)^2 + 4a^2})/2
\]

are parameterized by the acceleration, which can be extracted from the resonance frequencies measurements. In addition to sensing applications, the ability of the mechanical superheterodyne to convert low frequency, broad band, signals into high frequency resonant oscillations of the tilting mode is a beneficial feature in the field of energy harvesters [40]. Linearity of the mixing term in acceleration is especially attractive in this case, since the device can be operated at high substrate accelerations and therefore at high input powers.

To summarize, we believe that the architecture and the operational concept of the purely mechanical and therefore scalable, reliable and manufacturable heterodyne presented in this work, opens new intriguing possibilities in the realm of micro and nano mechanical signal processing devices, and may lead to better and more efficient designs of these systems and
their implementation in new applications.

ACKNOWLEDGEMENTS

The last author acknowledges support from the Henry and Dinah Krongold Chair of Microelectronics. Author R.R. acknowledges support by the National Science Foundation under grant number CMMI-1634664.

REFERENCES

Micromechanical superheterodyne and its use for low frequency vibration sensing. Supplementary materials.

Naftaly Krakover$^{1,*}$, Ronen Maimon$^{1,2,*}$, Tamar Tepper-Faran$^{2}$, Yuval Gerson$^{2}$, Richard Rand$^{3}$, and Slava Krylov$^{1}$

$^1$School of Mechanical Engineering, Faculty of Engineering, Tel Aviv University, Ramat Aviv 69978 Tel Aviv, Israel
$^2$MANOR A.D.T. Div., Rafael Advanced Defense Systems Ltd., Israel
$^3$Department of Mathematics and Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca NY 14853

$^*$These authors contributed equally to the work

Equations of motion in the presence of imperfections

To highlight the influence of possible imperfections on the device response, we consider the case when the mass in not perfectly aligned with the frame, Fig. S.1. A small offset $\hat{\delta} \ll \hat{h}$ is assumed to exist between the torsional ($x$) axis of the frame and the centroid $C$ of the mass. This offset affects both inertial forces, acting on the device due to the accelerated motion of the substrate, and the electrostatic forces applied to the mass by the comb drive transducer. It is instructive to consider and analyze the influence of these two effects separately.

Inertial forces

Consider first the frame-mass system inertially excited by an oscillating substrate, Fig. S.1. The coordinates $\hat{y}_C$, $\hat{z}_C$ of the mass centroid in the inertial reference frame (with the origin located at the tilting axis, at the mid height of the frame) are

$$\hat{y}_C = \hat{u} \cos(\theta) + \hat{\delta} \sin(\theta) \quad \hat{z}_C = \hat{\delta} + \hat{u} \sin(\theta) - \hat{\delta} \cos(\theta) \quad (S.1)$$
Here $\hat{z}(\hat{t})$ is the displacement of the substrate in the vertical Z-direction, $\hat{u}(\hat{t})$ is the displacement of the mass within the plane of the tilting frame and $\theta(\hat{t})$ is the tilting angle (see also Fig. 2 in the paper). The kinetic and the elastic energies of the device composed of the mass and of the tilting frame are

$$T = \frac{1}{2} (I_F + I_M) \left( \frac{d\theta}{dt} \right)^2 + \frac{1}{2} M \left[ \left( \frac{d\hat{u}}{dt} \right)^2 + \left( \frac{d\hat{z}}{dt} \right)^2 \right]$$ (S.2)

$$U = \frac{1}{2} k_\theta \theta^2 + \frac{1}{2} k_u \hat{u}^2$$ (S.3)

Here $M$ is the proof mass, and $I_F$ and $I_M$ are, respectively, the frame and the mass moments of inertia around their central axes; $k_\theta$ is the stiffness of the torsional spring attaching the frame to the substrate and $k_u$ is the stiffness of the spring attaching the mass to the frame. By substituting Eq. (S.1) into Eq. (S.2) and by assuming the small tilting angle such that $\sin(\theta) \approx \theta$, $\cos(\theta) \approx 1$ we obtain

$$T = \frac{1}{2} \left[ I_F + I_M + M(\hat{u}^2 + \hat{z}^2) \right] \left( \frac{d\theta}{dt} \right)^2 + \frac{1}{2} M \left[ \left( \frac{d\hat{u}}{dt} \right)^2 + \left( \frac{d\hat{z}}{dt} \right)^2 \right] + M \left[ \theta \left( \frac{d\hat{u}}{dt} \right) + (\hat{u} + \hat{\delta}\theta) \left( \frac{d\theta}{dt} \right) \right] \left( \frac{d\hat{z}}{dt} \right) + M \hat{\delta} \left( \frac{d\hat{u}}{dt} \right) \left( \frac{d\theta}{dt} \right)$$ (S.4)

Substituting of the kinetic, Eq. (S.4), and the strain Eq. (S.3) energies along with the Rayleigh dissipation function $\mathcal{R} = c_\theta (d\theta/d\hat{t})^2/2 + c_u (d\hat{u}/d\hat{t})^2/2$ (where $c_\theta$ and $c_u$ are the viscous damping coefficient associated with the tilting frame and the translational motion) into the Lagrange’s equations of the second kind yields the equation of motion of the inertially excited system

$$M \frac{d^2\hat{u}}{d\hat{t}^2} + c_u \frac{d\hat{u}}{d\hat{t}} + \left[ k_u - M \left( \frac{d\theta}{d\hat{t}} \right)^2 \right] \hat{u} = -M\theta \frac{d^2\hat{z}}{d\hat{t}^2} - M\hat{\delta} \frac{d^2\theta}{d\hat{t}^2}$$ (S.5)

$$\left[ I_F + I_M + M(\hat{u}^2 + \hat{z}^2) \right] \frac{d^2\theta}{d\hat{t}^2} + 2M\hat{u} \left( \frac{d\hat{u}}{d\hat{t}} \right) \left( \frac{d\theta}{d\hat{t}} \right) + c_\theta \frac{d\theta}{d\hat{t}} + k_\theta \theta = -M(\hat{u} + \hat{\delta}\theta) \frac{d^2\hat{z}}{d\hat{t}^2} - M\hat{\delta} \frac{d^2\theta}{d\hat{t}^2}$$ (S.6)

Since $\hat{\delta} \ll \hat{u}$, we neglect the terms containing $\hat{\delta}^2$ and $\hat{\delta}\theta$ in Eq. (S.6) and obtain

$$M \frac{d^2\hat{u}}{d\hat{t}^2} + c_u \frac{d\hat{u}}{d\hat{t}} + \left[ k_u - M \left( \frac{d\theta}{d\hat{t}} \right)^2 \right] \hat{u} = -M\theta \frac{d^2\hat{z}}{d\hat{t}^2} - M\hat{\delta} \frac{d^2\theta}{d\hat{t}^2}$$ (S.7)

$$\left[ I_F + I_M + M\hat{u}^2 \right] \frac{d^2\theta}{d\hat{t}^2} + 2M\hat{u} \left( \frac{d\hat{u}}{d\hat{t}} \right) \left( \frac{d\theta}{d\hat{t}} \right) + c_\theta \frac{d\theta}{d\hat{t}} + k_\theta \theta = -M\hat{u} \frac{d^2\hat{z}}{d\hat{t}^2} - M\hat{\delta} \frac{d^2\theta}{d\hat{t}^2}$$ (S.8)

**Electrostatic forces**

The electrostatic co-energy of the left and right electrostatic comb drive transducers is given by the expression

$$W_e = -\frac{C_R V_R^2}{2} - \frac{C_L V_L^2}{2}$$ (S.9)

Here $V_R$ and $V_L$ are the voltages applied to the right and to the left transducer, respectively.

To calculate the corresponding capacitances $C_R$ and $C_L$ the geometry of the comb drive actuator should be specified. Consider first the case when the fixed and movable combs of the transducers are perfectly aligned in the initial configuration, Fig. S.1. Since $\theta \ll 1$ the fixed and movable combs are assumed to remain parallel to the y-axis in the deformed configuration. As a result the overlap area between the fingers remains rectangular. It is to say the vertical displacement $w$ of the movable comb due to the frame rotation $\theta$ is assumed to be solely in the vertical z-direction. This displacement is calculated at the centroid of the overlap area between the fixed and the movable combs. Under these assumptions, the capacitances $C_R$ and $C_L$ are given by the expressions

$$C_R = \frac{2N\varepsilon_0}{g_0} (L_e + \hat{u}) \left( \hat{h} - \left( \hat{\gamma}_e + \frac{\hat{u}}{2} \right) \theta \right)$$ (S.10)

$$C_L = \frac{2N\varepsilon_0}{g_0} (L_e - \hat{u}) \left( \hat{h} - \left( \hat{\gamma}_e - \frac{\hat{u}}{2} \right) \theta \right)$$ (S.11)
Here $\varepsilon_0$ is the permittivity of vacuum, $g_0$ is the distance between the fixed and the movable fingers of the comb drive actuator, $L_e$ is the overlap length between the electrodes, $N$ is the number of the combs, Fig. S.1. The actuating force $F_u$ and the torque $M_\theta$ are obtained by calculating the derivatives of the electrostatic co-energy $W_c$ with respect to the corresponding degrees of freedom

$$F_u = \frac{V_R^2}{2} \left( \frac{\partial C_R}{\partial \hat{y}} \right) + \frac{V_L^2}{2} \left( \frac{\partial C_L}{\partial \hat{y}} \right)$$  \hspace{1cm} (S.12)

$$M_\theta = \frac{V_R^2}{2} \left( \frac{\partial C_R}{\partial \theta} \right) + \frac{V_L^2}{2} \left( \frac{\partial C_L}{\partial \theta} \right)$$  \hspace{1cm} (S.13)

After substituting Eqs. (S.10) and (S.11) into Eqs. (S.12) and (S.13) we obtain

$$F_u = \frac{N\varepsilon_0 V_R^2}{g_0} \left( \hat{h} - (2\hat{y}_e + L_e + 2\hat{u}) \frac{\theta}{2} \right) - \frac{N\varepsilon_0 V_L^2}{g_0} \left( \hat{h} - (2\hat{y}_e + L_e - 2\hat{u}) \frac{\theta}{2} \right)$$  \hspace{1cm} (S.14)

$$M_\theta = -\frac{N\varepsilon_0 V_R^2}{g_0} (L_e + \hat{u}) \left( \hat{y}_e + \frac{\hat{u}}{2} \right) - \frac{N\varepsilon_0 V_L^2}{g_0} (L_e - \hat{u}) \left( \hat{y}_e - \frac{\hat{u}}{2} \right)$$  \hspace{1cm} (S.15)

In accordance with Eq. (S.15), the electrostatic torque acting on the frame is $\theta$-independent. However, as was shown by many authors \cite{1-3}, at the small angles this torque can be significantly smaller than the value obtained using Eq. (S.15). At small $\theta$ the torque can be approximated by a linear function. Moreover, the fixed and the movable combs are never perfectly aligned in the initial configuration and imperfections, which inevitably exist, may give rise to a small force in the $z$ direction even at $\theta = 0$. These imperfections may arise due to the fabrication tolerances, residual stress gradients within the device layer, trapezoidal rather than rectangular fingers cross section, or parasitic fringing fields emerging from the substrate or side walls of the opening etched in the substrate. Since in the present work we are mainly focused on the dynamics of the device at small angles $\theta \ll 1$, thin black line and dashed lines represent the torque provided by the right (R) and left (L) transducers, respectively.

**Figure S.2.** (a) Schematic representation of the dependence between the vertical electrostatic force $F_z$ and the comb deflection $w$ in the vertical $z$ direction (thin gray line). Thick black line depicts the approximation by a linear function, circle corresponds to an initial value of $F_z$ in the case of the movable comb offset $\hat{u}$. (b) Torque $M_x$ applied by the comb drive transducer (thin gray line). Thick solid line corresponds to the linear approximation at $\theta \ll 1$, thin black line and dashed lines represent the torque provided by the right (R) and left (L) transducers, respectively.

In view of the aforementioned, the modified capacitances of the right and of the left transducers are given by the expressions

$$C_R = \frac{2N\varepsilon_0}{g_0} (L_e + \hat{u}) \left( \hat{h} + \eta \left( \hat{y}_e + \frac{\hat{u}}{2} \right) \theta - \nu \left( \hat{y}_e + \frac{\hat{u}}{2} \right)^2 \frac{\theta^2}{2} \right)$$  \hspace{1cm} (S.16)

$$C_L = \frac{2N\varepsilon_0}{g_0} (L_e - \hat{u}) \left( \hat{h} - \eta \left( \hat{y}_e - \frac{\hat{u}}{2} \right) \theta - \nu \left( \hat{y}_e - \frac{\hat{u}}{2} \right)^2 \frac{\theta^2}{2} \right)$$  \hspace{1cm} (S.17)

Here the parameter $\nu$ depends on the electrodes geometry, mainly on the on the comb’s width and on the ratio $g_0/\hat{h}$, while $\eta$ is dictated by the initial imperfection $\hat{u}$. In general, $M_\theta$ reaches the maximal value given by Eqs. (S.15) at the angle $\theta$...
corresponding to $\hat{v} \approx \hat{g}_0$. Thus the parameters $\eta$ and $\hat{V}$ can be approximated as $\eta \approx \hat{\delta}/\hat{g}_0 \ll 1$, $\hat{v} \approx 1/\hat{g}_0 \, ^3$. Note that in accordance with Eq. (S.16), (S.17) we neglect the influence of the offset $\hat{\delta}$ on the axial displacement of the mass, i.e., we assume $\hat{u} + \hat{\delta}\theta \approx \hat{u}$.

Substituting of the modified capacitances Eqs. (S.16) and (S.17) into Eqs. (S.12) and (S.13) yields

$$F_u = \frac{N\epsilon_0 V_R^2}{\hat{g}_0} \left[ \hat{h} + \eta \left( \hat{y}_e + \frac{\hat{u}}{2} \right) \theta - \hat{\nu} \left( \hat{y}_e + \frac{\hat{u}}{2} \right)^2 \right] + \frac{N\epsilon_0 V^2}{\hat{g}_0} \left[ -\hat{h} + \eta \left( \hat{y}_e - \frac{\hat{u}}{2} \right) \theta + \hat{\nu} \left( \hat{y}_e - \frac{\hat{u}}{2} \right)^2 \right] \left( \hat{L}_e + \hat{\delta} \theta \right)$$

(S.18)

$$M_\theta = \frac{N\epsilon_0 V_R^2}{\hat{g}_0} \left( \hat{L}_e + \hat{\delta} \theta \right) \left[ \eta \left( \hat{y}_e + \frac{\hat{u}}{2} \right) - \hat{\nu} \left( \hat{y}_e + \frac{\hat{u}}{2} \right)^2 \right] - \frac{N\epsilon_0 V^2}{\hat{g}_0} \left( \hat{L}_e - \hat{\delta} \theta \right) \left[ \eta \left( \hat{y}_e - \frac{\hat{u}}{2} \right) + \hat{\nu} \left( \hat{y}_e - \frac{\hat{u}}{2} \right)^2 \right]$$

(S.19)

It is more convenient to re-write Eqs. (S.18), (S.19) in the form

$$F = \frac{N\epsilon_0 (V_R^2 - V^2)}{\hat{g}_0} \left[ \hat{h} + \eta \hat{\alpha} \theta - \hat{\nu} \left( \hat{y}_e + \hat{L}_e \right) + \frac{3\hat{g}^2}{4} \right] \left( \hat{\eta} \hat{\nu} \right) \left( \hat{y}_e + \hat{L}_e \right) \left( \frac{\hat{g}^2}{4} \right)$$

(S.20)

$$M_\theta = \frac{N\epsilon_0 (V_R^2 - V^2)}{\hat{g}_0} \left[ \eta \left( \hat{L}_e \hat{y}_e + \hat{u}^2 \right) - \hat{\nu} \hat{\alpha} \theta \left( \hat{y}_e + \hat{L}_e \right) + \frac{\hat{g}^2}{4} \right] + \frac{N\epsilon_0 (V_R^2 + V^2)}{\hat{g}_0} \left[ \eta \left( \hat{y}_e + \hat{L}_e \right) + \frac{\hat{g}^2}{4} \right] + \frac{N\epsilon_0 (V_R^2 + V^2)}{\hat{g}_0} \left[ \eta \left( \hat{y}_e + \hat{L}_e \right) + \frac{\hat{g}^2}{4} \right]$$

(S.21)

Since in our case $\hat{u} \ll \hat{L}_e < \hat{y}_e$, Eqs. (S.21), (S.20) can be further simplified by neglecting the terms containing $\hat{u}^2$. For the same reason $\hat{y}_e \hat{\alpha} \theta \ll \eta \hat{y}_e$. Moreover, in accordance with our assumptions, the z-displacements of the combs are much smaller than the layer thickness and the distance between the combs, i.e., $\hat{\delta} \theta < \hat{y}_e \hat{\theta} \ll \hat{g}_0 < \hat{h}$. Thus, the terms in Eq. (S.20) containing $\eta \hat{\alpha} \theta$ and $\hat{\nu}$ can be neglected when compared to $\hat{h}$. Also, we neglect the term containing $\hat{\theta}^2$ in Eq. (S.20). As a result, we obtain the simplified expressions for the electrostatic force and torque incorporating only the direct driving and linear coupling terms

$$F_u = \frac{N\epsilon_0 h}{\hat{g}_0} (V_R^2 - V^2) + \frac{N\epsilon_0 (V_R^2 + V^2)}{\hat{g}_0} \left[ \eta \left( \hat{y}_e + \frac{\hat{L}_e}{2} \right) \theta \right]$$

(S.23)

$$M_\theta = \frac{N\epsilon_0 h}{\hat{g}_0} (V_R^2 - V^2) \eta \hat{y}_e \hat{L}_e + \frac{N\epsilon_0 (V_R^2 + V^2)}{\hat{g}_0} \left[ \eta \left( \hat{y}_e + \frac{\hat{L}_e}{2} \right) \theta - \hat{\nu} \hat{L}_e \hat{y}_e \theta \right]$$

(S.24)

**Governing equations in the presence of imperfections**

The equations governing the dynamics of the system are obtained by combining the contributions of the inertial, Eqs. (S.8), (S.8), and electrostatic, (S.20), (S.24), excitations

$$M \frac{d^2\hat{u}}{dt^2} + c_v \frac{d\hat{u}}{dt} + \left[ k_v - M \left( \frac{d\theta}{dt} \right)^2 \right] \hat{\alpha} = -M\theta \hat{\alpha} - M\hat{\delta} \frac{d^2\theta}{dt^2} + \frac{N\epsilon_0 \hat{h}}{\hat{g}_0} (V_R^2 - V^2) + \frac{N\epsilon_0 (V_R^2 + V^2)}{\hat{g}_0} \eta \left( \hat{y}_e + \frac{\hat{L}_e}{2} \right) \theta$$

(S.25)

$$[I_f + I_M + M\hat{\alpha}^2] \frac{d^2\theta}{dt^2} + 2M\hat{\alpha} \left( \frac{d\hat{u}}{dt} \right) + \eta \left( \frac{d\theta}{dt} \right) + c_\theta \frac{d\theta}{dt} + k_\theta \theta = -M\hat{\alpha} \hat{\alpha} - M\hat{\delta} \frac{d^2\hat{\alpha}}{dt^2} + \frac{N\epsilon_0 (V_R^2 - V^2)}{\hat{g}_0} \eta \hat{y}_e \hat{L}_e + \frac{N\epsilon_0 (V_R^2 + V^2)}{\hat{g}_0} \left[ \eta \left( \hat{y}_e + \frac{\hat{L}_e}{2} \right) \theta - \hat{\nu} \hat{L}_e \hat{y}_e \theta \right]$$

(S.26)

where $\hat{\alpha}(t)$ is the acceleration of the substrate.
It is more convenient to re-write the equations (S.26) and (S.25) in the non-dimensional form

\[
\ddot{u} + 2\zeta_0 \omega_0 \dot{u} + (\omega_0^2 - \dot{\theta}^2)u = -a_S \theta - \delta \dot{\theta} + \beta h (V_R^2 - V_L^2) + \beta (V_R^2 + V_L^2) \eta \left( y_e + \frac{L_e}{2} \right) \theta
\]

(S.27)

\[
(1 + u^2) \dot{\theta} + 2u \dot{\theta} + 2\zeta_0 \theta + \theta = -a_S u - \delta u + \beta (V_R^2 - V_L^2) \eta y_e L_e + \beta (V_R^2 + V_L^2) \left[ \eta \left( y_e + \frac{L_e}{2} \right) u - v L_e y_e^2 \dot{\theta} \right]
\]

(S.28)

The non-dimensional quantities appearing in Eqs. (S.28), (S.27) are defined by the expressions

\[
\begin{align*}
t &= \frac{\dot{t}}{\tau} \quad u = \frac{\dot{u}}{r} \quad y_e = \frac{\dot{y}_e}{r} \quad L_e = \frac{L_e}{r} \quad h = \frac{h}{r} \quad \delta = \frac{\delta}{r} \quad v = \frac{\nu}{r} \\
\omega_\alpha &= \frac{\dot{\omega}_\alpha}{\omega_0} \quad a_S = \frac{\dot{a}_S}{r \dot{\omega}_0} \quad \beta = \frac{N e_0 V_0^2 \omega_0^2}{g_0 k_a} \quad \zeta_0 = \frac{c_0}{2 \sqrt{k_0 (I_F + I_M)}} \quad \zeta_\alpha = \frac{c_u}{2 \sqrt{k_u M}}
\end{align*}
\]

(S.29)

Here \( \omega_\alpha = \sqrt{k_\alpha / M} \) is the mass frequency, \( \dot{\omega}_\alpha = \sqrt{k_\alpha / (I_F + I_M)} \) is the tilting frequency for the case of unmovable mass, and \( V_0 \) is the unit voltage. In accordance with Eq. (S.29) the frame’s gyration radius \( r = \sqrt{(I_F + I_M) / M} \) and the period of the free tilting vibrations \( \tau = 1 / \dot{\omega}_0 \) are used as units of length and time, respectively.

To highlight the origin of the coupling in the system under consideration along with the sources of excitation, is instructive to re-write Eqs. (S.28), (S.27) in the form

\[
\ddot{u} + 2\zeta_0 \omega_0 \dot{u} + (\omega_0^2 - \dot{\theta}^2)u + \left[ a_S - \beta \eta \left( y_e + \frac{L_e}{2} \right) (V_R^2 + V_L^2) \right] \theta + \delta u = \beta h (V_R^2 - V_L^2)
\]

(S.30)

\[
(1 + u^2) \dot{\theta} + 2u \dot{\theta} + 2\zeta_0 \theta + \left[ 1 + \beta v L_e y_e^2 V_R^2 + V_L^2 \right] \theta + \left[ a_S - \beta \eta \left( y_e + \frac{L_e}{2} \right) (V_R^2 + V_L^2) \right] u + \delta u = \beta \eta y_e L_e (V_R^2 - V_L^2)
\]

(S.31)

The acceleration of the substrate as well as the voltages applied to the right and left comb drive transducers are assumed to be harmonic functions

\[
a_S = a_0 \cos(\Omega t) \quad V_R = V_{dc} + V_{ac} \cos(\omega t) \quad V_L = V_{dc} - V_{ac} \cos(\omega t)
\]

(S.32)

where \( \Omega = \dot{\Omega} / \omega_\theta \) and \( \omega = \dot{\omega} / \omega_\theta \) are the non-dimensional frequencies of the substrate acceleration and of the voltage signal, respectively and \( a_0 = \dot{a}_0 / (r \dot{\omega}_0) \). Since \( V_R \) and \( V_L \) contain both steady (dc) and time varying (ac) components, the device is subjected to a two-frequency excitation

\[
V_R^2 - V_L^2 = 2 V_{dc} V_{ac} \cos(\omega t) \quad V_R^2 + V_L^2 = 2 V_{dc}^2 + V_{ac}^2 \cos(2 \omega t)
\]

(S.33)

By considering Eqs. (S.31), (S.30) one can notice that the equations are of the parametric nature.\(^4\) Time varying coefficients appear due to the variable moment of inertia associated with the tilting degree of freedom (the first two terms in Eqs. (S.31)) as well as due to the time-dependent electrostatic moments acting on the frame. Our analysis show that for the typical values of the actuation voltages, the mass displacement is small and the influence of the variable inertia is minor. The softening influence of the centrifugal terms \( \dot{\theta}^2 \) on the effective stiffness of the mass in Eq. (S.30) was found to be negligible as well.

The linearised counterpart of Eqs. (S.30), (S.31), obtained by neglecting the variable inertia and the centrifugal stiffness, can be conveniently written in the matrix form

\[
M \ddot{q} + C \dot{q} + K q = F
\]

(S.34)

Here \( q = \{u, \theta\}^T \) (where "T" denotes the matrix transpose) is the vector of the degrees of freedom. The mass, damping and stiffness matrices are given by the expressions

\[
M = \begin{pmatrix} 1 & \delta \\ \delta & 1 \end{pmatrix} \quad C = \begin{pmatrix} \zeta_0 \omega_0^2 & 0 \\ 0 & \zeta_\theta \end{pmatrix} \quad K = \begin{pmatrix} a_S - \beta \eta \left( y_e + \frac{L_e}{2} \right) (V_R^2 + V_L^2) & a_S - \beta \eta \left( y_e + \frac{L_e}{2} \right) (V_R^2 + V_L^2) \\ \beta v L_e y_e^2 V_R^2 + V_L^2 & 1 + \beta v L_e y_e^2 V_R^2 + V_L^2 \end{pmatrix}
\]

(S.35)

The force vector containing the direct excitation terms is

\[
F = \left\{ \begin{array}{l} \beta h (V_R^2 - V_L^2) \\ \beta \eta y_e L_e (V_R^2 - V_L^2) \end{array} \right\}
\]

(S.36)
Equation (S.35) shows that both inertial and stiffness couplings between the mass translational motion and the frame tilting are present. Our numerical results indicate that, since $\delta \ll 1$, the inertial coupling in the mass matrix is not pronounced and can be neglected. Moreover, in our experiments the excitation voltages applied to the electrodes were small enough to not surpass the parametric resonance threshold. As a result, parametric resonances were not observed in the experiments and only linear Lorentzian spectral responses were registered. However, while measuring the tilting response of the frame, resonant peaks were observed at the frequencies close to $\omega_\theta$ and $\omega_\nu$. This result indicates that the coupling between the degrees of freedom, and therefore the imperfections, cannot be disregarded. In accordance with Eq. (S.33) the direct driving term proportional to $V_R^2 + V_L^2$ leads to the excitation of the device at the frequency of $2\omega$. Since in our experiments the device was driven at the frequency $\omega$ in the vicinity of the resonant frequencies, namely, $\omega_\theta$ and $\omega_\nu$, the excitation at the twice of these frequencies does not lead to resonant excitation. On the other hand, by considering Eq. (S.30) one observes that the direct driving term $\beta h(V_R^2 - V_L^2)$ is dominant whereas the influence of the tilting DOF on the motion of the mass is minor. For this reason, we completely neglect the influence of the tilting DOF on the motion of the mass in Eq. (S.30), but preserve the frequency mixing term $a_Su$ and the direct driving term $\beta \eta L_e(V_R^2 - V_L^2)$ as well as the linear coupling term $\beta \eta (y_e + L_e/2)(V_R^2 + V_L^2)$ in Eq. (S.31).

In view of the aforesaid, by neglecting the inertial coupling, parametric terms and multifrequency excitation, probably the simplest version of Eqs. (S.30), (S.31), which capture the leading effects in the device’s behavior and which was used in the calculations, was obtained

$$\ddot{u} + 2\zeta u \dot{u} + \omega_u^2 u = 2\beta h V_d e \cos(\omega t)$$

$$\ddot{\theta} + 2\zeta \theta \dot{\theta} + \theta = \left(\beta \eta \left(y_e + \frac{L_e}{2}\right)(2V_d e + V_{ac}^2) - a_S\right) u + 2\beta \eta L_e V_{dc} V_{ac} \cos(\omega t)$$

### Finite element analysis results

The details of the FE model are presented in the Methods section of the manuscript. The four lowest natural modes of vibrations are shown in Fig. S.3. Two desired operational modes, namely the proof masses in-phase translation mode, distinguished by the mass vibrations within the plane of the frame, and the tilting mode, are shown in Fig. S.3(a) and 4(c), respectively. The corresponding frequencies were 4494 Hz and 6453 Hz. The masses anti-phase translational mode and the out-of-plane mode are depicted in Fig. S.3(b) and (d), respectively. The frequencies were 4594 Hz and 6453 Hz.

The finite elements modeling was used as a tool for evaluation of the design parameters. The geometric parameters of the device and mainly the suspension beams length and width, were chosen to provide sufficient separation between the desired working modes frequencies (in-phase translational and tilting) and undesired modes frequencies (anti-phase translational and out-of-plane). The smallest split of 145 Hz is between the in-phase and anti-phase (tuning fork) translational modes.

### References


Figure S.3. Four lowest vibrational modes of the device: (a) in-phase translational mode, $f_1 = 4449$ Hz; (b) anti-phase translational mode, $f_2 = 4594$ Hz. In both (a) and (b) colors indicate the non normalized displacements in x direction. (c) tilting mode, $f_3 = 4920$ Hz and (d) out-of-plane mode, $f_4 = 6453$. In (c) and (d) colors indicate the non normalized displacements in z direction.