[2] A. T. WINFREE, *The Geometry of Biological Time*, Springer-Verlag, New York, 2001.

ROBERT E. O'MALLEY, JR.
University of Washington

Chaotic Synchronization: Applications to Living Systems. *By Erik Mosekilde, Yuri Maistrenko, and Dmitry Postnov.* World Scientific, River Edge, NJ, 2002. $76.00. xii+428 pp., hardcover. ISBN 981-02-4789-3.

This book deals with systems of coupled oscillators which are exhibiting chaotic behavior. The main concern is finding for which parameters the individual oscillators remain synchronized (while each is behaving chaotically), and the bifurcations which lead to this synchronization.

Here are some examples of the types of systems considered: In Chapters 2 and 3 a system of two coupled logistic maps are studied:

$$(1)\quad x_{n+1} = ax_n(1-x_n) + \epsilon(y_n - x_n),$$

$$(2)\quad y_{n+1} = ay_n(1-y_n) + \epsilon(x_n - y_n).$$

In Chapter 4 a system of two coupled Rössler oscillators is studied:

$$(3)\qquad \dot{x} = f(x) + C(y-x),$$

$$(4)\qquad \dot{y} = f(y) + C(x-y),$$

where $x = (x_1, x_2, x_3)^T$ and where

$$(5)\quad f(x) = \begin{pmatrix} -x_1 - x_3 \\ x_1 + ax_2 \\ b + x_3(x_1 - c) \end{pmatrix}, \quad C = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}.$$

For these examples, attention is given to the stability of chaotic motions on the *synchronization manifold* $y = x$. The book utilizes a variety of relatively new concepts to describe the resulting dynamics: *blowout bifurcations, riddling bifurcations*, and *riddled basins of attraction* [1, 2]. In order to give the reader a feeling for the flavor of the book, I offer the following explanation of these concepts. Suppose that the motion on the synchronization manifold is that of a chaotic attractor. Let $\lambda_\perp$ be the largest Lyapunov exponent which represents motion transverse to the synchronization manifold. Then the transition at which $\lambda_\perp$ changes sign is referred to as a *blowout bifurcation.* If instead the motion on the synchronization manifold consists of a periodic motion (which is embedded in the synchronized chaotic attractor), then the associated value of $\lambda_\perp$ will in general depend upon the period of the motion. Now consider the set of all such periodic motions. The *riddling bifurcation* corresponds to the parameter at which the first of this set loses its transverse stability. For parameters which lie between the blowout bifurcation and the riddling bifurcation, the basin of attraction of the synchronized chaotic motion in phase space is found to be fractal and is referred to as a *riddled basin of attraction.*

In Chapters 1 and 6 the dynamics of a forced Rössler oscillator is studied:

$$(6)\qquad \begin{aligned} \dot{x} &= -\omega y - z + K \sin \omega_f t, \\ \dot{y} &= \omega x + \alpha y, \\ \dot{z} &= \beta + z(x - \mu). \end{aligned}$$

For fixed values of α, β, μ, and ω, the unforced $K = 0$ system may exhibit chaos. For some parameters this will be characterized by a strong peak in the power spectrum. When the system is forced with fixed forcing frequency ω_f, it is found that as ω is varied, there is a plateau of 1:1 locking in which the peak frequency of the spectrum is equal to the forcing frequency.

Chapter 6 contains a discussion of the dynamics of a system of N phase-only oscillators:

$$(7)\qquad \begin{aligned} \phi^i_{n+1} &= \phi^i_n + \omega^i \\ &\quad + K \sin\left(2\pi \sum_{j=1}^{N} (\phi^j_n - \phi^i_n)\right) \bmod 1, \\ &\quad i = 1, \ldots, N, \end{aligned}$$

where ϕ^i_n is the phase of the ith oscillator at time n and ω^i is its uncoupled frequency. A detailed treatment for $N = 3$ and $K < 0$ is presented involving a complicated series of bifurcations in which nonattractive fractal sets (*chaotic saddles*) coexist with chaotic attractors. We are told that the bifurcation sequence includes a transition to *hyperchaos,* a term which evidently corresponds to situations in which there are at least two positive Lyapunov exponents. I was not familiar with this term and sought a definition by looking in the index. Although there were four pages listed as referring to hyperchaos, I couldn't find any reference to it on three of the four pages listed.

In addition to the treatment of abstract systems of oscillators, such as those described above, about 100 pages, or roughly one-quarter of the book, are devoted to applications, mainly to biological systems. These include Chapter 5 on "Coupled Pancreatic Cells," Chapter 7 on "Population Dynamic Systems" (based on a bacteria-virus model), and Chapter 9 on "Interacting Nephrons" (kidney units). In addition, Chapter 1 includes a discussion of "Mode-Locking of Endogenous Economic Cycles." Each of the three chapters on biological applications begins with a section introducing the biology. Then an investigation of the dynamics of a single unit is presented. In the case of bursting pancreatic β-cells, this consists of a system of three first order ordinary differential equations (ODEs). In the application to bacterial cells attacked by a virus, the model involves a system of four first order ODEs. The model of a single nephron consists of a system of six first order ODEs. In each case the model of the individual unit is shown to exhibit chaos in some parameter range. Each of the three applications is then treated as a coupled system, respectively, as coupled chaotically spiking pancreatic cells, two cascaded microbiological reaction systems, and two coupled nephrons. In each case attention is focused on chaotic synchronization.

Each chapter has an extensive bibliography, and the book has a subject index. There is no author index, however.

Is this the book for you? If you are looking for a book with theorems or asymptotic expansions, the answer is no. Virtually all the results are obtained by numerical simulation. This is to be expected because of the intractability of the governing equations. If you are looking for a textbook with which to teach a graduate course on chaos, the answer is also no. This book is written for a reader who is familiar with standard treatments of nonlinear dynamics and chaos, for example, at the level of Jackson [3, 4]. However, if you are a researcher in nonlinear dynamics and you want to learn about the terminology and basic ideas of chaotic synchronization, together with an extensive list of relevant research papers, and including some interesting biological applications, then this book may be for you.

REFERENCES

[1] J. C. Alexander, J. A. Yorke, Z. You, and I. Kan, *Riddled Basins*, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 2 (1992), pp. 795–813.

[2] E. Ott and J. C. Sommerer, *Blowout Bifurcations: The Occurrence of Riddled Basins and On-Off Intermittency*, Phys. Lett. A, 188 (1994), pp. 39–47.

[3] E. A. Jackson, *Perspectives of Nonlinear Dynamics*, Vol. 1, Cambridge University Press, Cambridge, UK, 1989.

[4] E. A. Jackson, *Perspectives of Nonlinear Dynamics*, Vol. 2, Cambridge University Press, Cambridge, UK, 1990.

R. H. Rand
Cornell University

Linear Algebra: A Pure Mathematical Approach. *By Harvey E. Rose.* Birkhäuser-Verlag, Basel, 2002. $59.95. xiv+250 pp., hardcover. ISBN 3-7643-6905-1.

The mathematics book market is full of linear algebra texts, and new ones keep appearing frequently. No wonder: Second only to calculus, linear algebra is a staple of undergraduate mathematical offerings at many colleges. However, notwithstanding fine pedagogical qualities of many undergraduate texts in linear algebra and their obvious usefulness in a classroom, there is little they can offer to a more advanced student or to a working mathematician (in any specialty).

There is another line of linear algebra books, not nearly as numerous as undergraduate texts. These books, often written as textbooks for a second undergraduate or a graduate course in linear algebra, retain (more or less) the "general purpose" outlook of undergraduate texts, but aim at a more mathematically mature audience. As a result, the material of these books, which